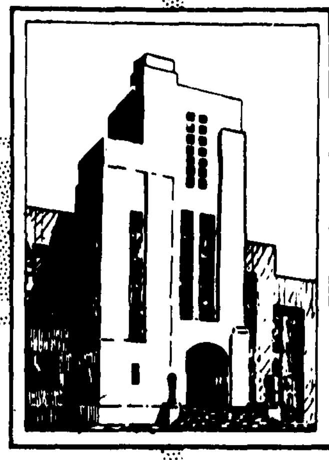

DEPARTMENT OF THE NAVY
DAVID TAYLOR MODEL BASIN

HYDROMECHANICS

AERODYNAMICS

STRUCTURAL MECHANICS

APPLIED MATHEMATICS

AXISYMMETRIC ELASTIC DEFORMATIONS AND STRESSES IN A WEB-STIFFENED SANDWICH CYLINDER UNDER EXTERNAL HYDROSTATIC PRESSURE

by

John G. Pulos

STRUCTURAL MECHANICS LABORATORY

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TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

NOTATION

$a_i, b_i, c_i, d_i, f_i, g_i$ $\bar{a}_i, \bar{b}_i, \bar{c}_i, \bar{d}_i, \bar{f}_i, \bar{g}_i$	Coefficients representing edge rotation and displacement per unit edge or surface load for shell elements of short length
$D_i = \dfrac{E^s h_i^3}{12(1-\nu^2)}$	Flexural rigidity of shells
E^A, E^s	Young's modulus of annulus and shell materials, respectively
H_i, H_j	Discontinuity shearing forces normal to axis of symmetry
h_i	Shell thickness
l	Length of shell element between stiffeners
M_i, M_j	Discontinuity bending moments in a meridional plane
P_i	Axial stress-forces due to axial portion of p
p	Hydrostatic pressure
R_i, R_j	Radial distances from axis of symmetry
r	Variable radial distance from axis of symmetry
w_i^A	Radial displacement of annulus edges
w_i^s	Radial displacement of shells
x	Axial coordinate taken along shell element
β_i	$= \dfrac{\sqrt[4]{3(1-\nu^2)}}{\sqrt{R_i h_i}}$
ϵ	Strain
θ_i^s	Axial rotation of shells
$\left.\begin{matrix}\Lambda^{[1]}, \Lambda^{[2]}, \Lambda^{[3]} \\ \Lambda^{[4]}, \Lambda^{[5]}, \Lambda^{[6]}\end{matrix}\right\}$	Lambda functions defining edge effects and interaction of edge effects for shell elements of short length
ν	Poisson's ratio
σ	Stress

ABSTRACT

A theoretical analysis of the axisymmetric elastic deformations and stresses in a web-stiffened sandwich cylindrical shell structure under external hydrostatic pressure is presented. The solution is based on the use of edge coefficients for plate and shell elements of finite length, and includes the computation of the edge forces and moments arising at the common junctures of these elements.

Equations are given for computing numerically the longitudinal and circumferential stresses in the two coaxial cylindrical shells and the radial and tangential stresses in the web stiffeners between the two shells.

No consideration was given to the discontinuity effects arising from rigid or elastic restraints afforded by contiguous bulkhead or adjacent shell structures. Thus, the analysis presented herein is applicable only to a typical bay of a web-stiffened sandwich cylinder of long length.

A numerical example is presented to illustrate the use of the equations developed in this report.

INTRODUCTION

The David Taylor Model Basin, under initial sponsorship by the Office of Naval Research and later continuance by the Bureau of Ships, has been investigating the feasibility of sandwich-type construction for pressure hull application. Results of exploratory experimental studies carried out under this program[1] have shown that in certain ranges of geometry strength-weight advantages on the order of 20 to 25 percent higher can be realized with sandwich designs over the conventional ring-stiffened cylindrical configuration. These results were obtained from model tests of sandwich-type cylinders having "hard" cores; i.e., the cores were capable of developing high compressive strengths in addition to transmitting the pressure loading by shear from the outer to the inner shell.

At the time these sandwich cylinders were conceived, no formulas were available on which to base an optimum design; merely intuition and engineering judgment were resorted to for proportioning the structural elements. Concurrently with the experimental program, analytical studies were initiated to develop rational formulas based on thin-shell theory for predicting the elastic deformations and stresses in the structural elements of such sandwich-type cylinders.

In this report, equations are developed for carrying out a complete stress analysis of a typical portion of a web-stiffened sandwich cylinder under external hydrostatic pressure.

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[1] References are listed on page 37.

The method is based on the use of edge coefficients of plate and shell elements of finite length, and satisfaction of force and moment equilibrium and compatibility of deformations at the common junctures of the elements comprising the structure. Expressions for edge coefficients of cylindrical shells of short length are developed in Appendix A.

GENERAL CONSIDERATIONS

Methods of analysis based on the use of edge coefficients have found wide application in studying stresses and deformations in complex structures composed of ring, plate, and shell elements.[2–5] The underlying concept in this type of analysis is that a complex physical structure can be broken down into identifiable components for which mathematical solutions exist or can be found readily. The deformations occurring in each structural element are determined in terms of unknown forces and moments assumed to exist at the junctures common to these elements. Conditions of equilibrium and compatibility are then satisfied at each of the junctures, thus permitting determination of the redundant forces and moments. With this information, a complete stress analysis for each structural component can then be performed.

The present problem of the stresses in a web-stiffened sandwich cylinder subjected to hydrostatic pressure, shown in Figure 1, can be solved rather conveniently by the use of edge coefficients. The identifiable structural elements in this case are two coaxial cylindrical

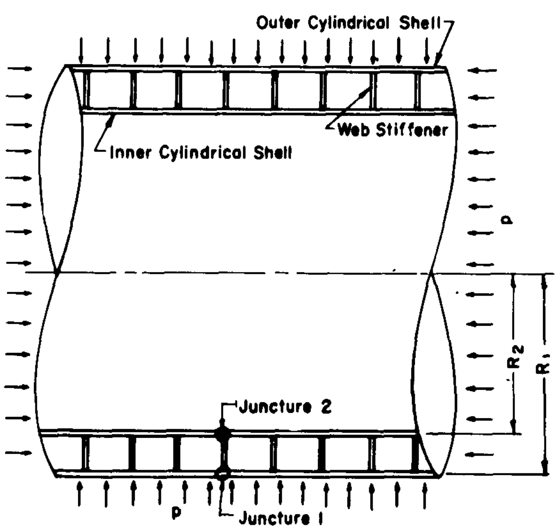
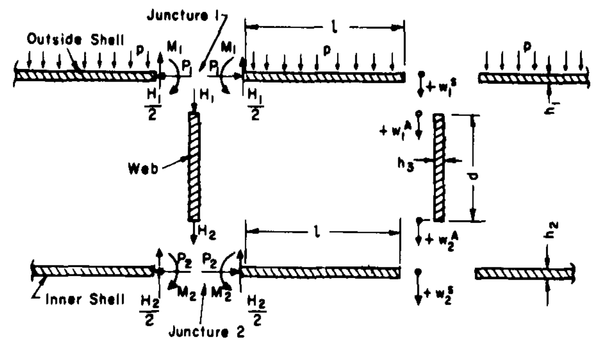


Figure 1 – Web-Stiffened Sandwich Cylinder Subjected to External Hydrostatic Pressure

shells, one subjected to radial pressure and an end load and the other to an end load only, and annular discs subjected to radial loads on the two circular boundaries. The webs or annular discs act as the connecting and stiffening members to the two shells. A free-body diagram showing the breakdown of the physical structure to its component parts, together with appropriate, but as yet unknown, edge forces and moments, is presented in Figure 2.

Figure 2 – Free-Body Diagram Showing Forces and Moments Acting on Shell and Web Elements of Web-Stiffened Sandwich Cylinder



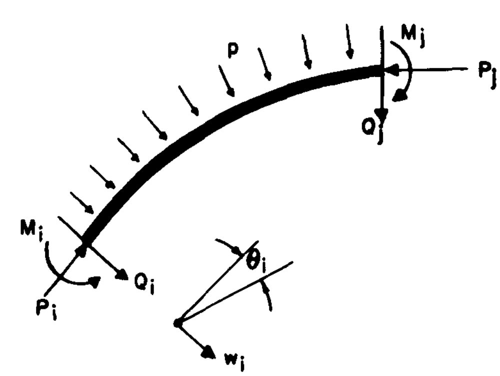
Following the method of References 2, 3, and 4, the deformations occurring at the edges of a shell element of general meridional shape can, by simple superposition, be written in terms of the unknown edge forces and edge moments and known applied loading as follows:

$$w_i = d_i M_i + g_i Q_i + f_i' P_i + f_i'' p_i + f_i''' P_i + d_{ij} M_j + g_{ij} Q_j + f_{ij}' P_j \qquad [1]$$

$$\theta_i = a_i M_i + b_i Q_i + c_i' P_i + c_i'' p + c_i''' P_i + a_{ij} M_j + b_{ij} Q_j + c_{ij}' P_j \qquad [2]$$

where the coefficients a_i, b_i, . . . f_i''' are the amount of transverse deflection or meridional rotation, as the case may be, per unit bending moment, shearing force, axial force, or surface pressure loading, as shown in Figure 3. The coefficients with the double subscripts, i.e.,

Figure 3 – Shell Element of Arbitrary Meridional Shape Subjected to Edge Moments, Shears, Forces, and Surface Loading



a_{ij}, b_{ij}, . . . f_{ij}', are the interaction coefficients which reflect the deformations at edge "i" due to forces and moments at edge "j." By replacing $i \rightarrow j$ and $j \rightarrow i$ in Equations [1] and [2], expressions for the deformations w_j and θ_j can be written immediately.

Note that the effect of the end load P on the deformations w_i and θ_i has been separated into three distinct components. The components denoted by the single-primed coefficients f_i' and c_i' are those due to bending effects. The same is true of the components associated with the coefficients f_{ij}' and c_{ij}', but these also reflect interaction influences. The components denoted by the triple-primed coefficients f_i''' and c_i''' are essentially Poisson effects on the membrane deformations.

For the specific problem of cylindrical shell elements symmetrically loaded, as shown in Figure 4 and considered in this report, Equations [1] and [2] become:

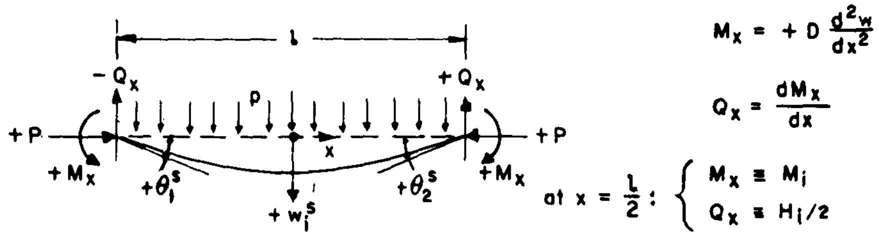


Figure 4 – Sign Convention for Cylindrical-Shell Element (Symmetric Case)

$$w_i^s = d_i M_i^s + g_i \frac{H_i^s}{2} + f_i' P_i^s + f_i'' p + f_i''' P_i^s + d_{ij} M_i^s + g_{ij} \frac{H_i^s}{2} + f_{ij}' P_i^s \qquad [3]$$

$$\theta_i^s = a_i M_i^s + b_i \frac{H_i^s}{2} + c_i' P_i^s + c_i'' p + c_i''' P_i^s + a_{ij} M_i^s + b_{ij} \frac{H_i^s}{2} + c_{ij}' P_i^s \qquad [4]$$

and similar expressions for w_j and θ_j, respectively. However, for the case of a cylinder some of the terms appearing in Equations [3] and [4] become zero; this will be shown later. In addition, for the pressure loading shown in Figure 1, where the inner cylindrical shell is not subjected to the radial pressure loading, those terms in Equations [3] and [4] that are multiplied by p will drop out when the deformations of the inner shell, i.e. $i = 2$, are considered.

Following the same technique employed for the shell elements, the deformations occurring at the edges of the circular annuli or web elements, as shown in Figure 5,

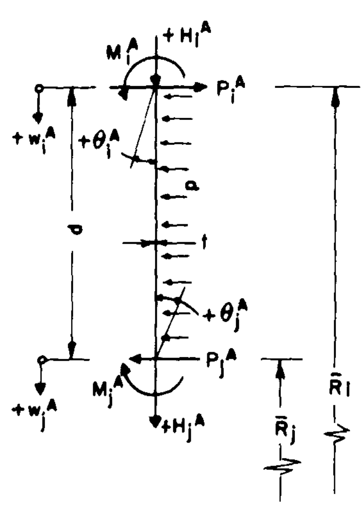


Figure 5 – Sign Convention for Web Element

can be written in terms of the unknown edge forces and moments and known applied loading as follows:

$$w_i^A = g_i^A H_i^A + g_{ij}^A H_j^A \qquad [5]$$

$$\theta_i^A = a_i^A M_i^A + c_i^A P_i^A + d_i^A p + a_{ij}^A M_j^A \qquad [6]$$

where it has been assumed that the edge moments M_i^A, axial thrusts P_i^A, and surface pressure p do not give rise to any radial displacement w in the plane of the circular annulus. Equations [5] and [6] give the deformations at boundary i of the circular annulus; the deformations at the other boundary, say j, can be obtained by replacing i by j and j by i in Equations [5] and [6]. The expression for the edge rotations θ_i^A and θ_j^A of the annulus are rather general to include the case in which the sandwich void between the two cylinders may become pressurized. This problem will not be considered in this report. In a later section, it will be shown that due to symmetry the edge rotations of the annulus are zero. Furthermore, for the particular case of pressure loading shown in Figure 1 and considered in detail later including a numerical example, not only are the edge rotations θ_i^A and θ_j^A equal to zero but every term in expressions [6] is zero. In such a case it is tacitly assumed that the web stiffeners act only to resist hoop compression and do not act in the sense of a circular plate to resist bending due to edge moments and edge shears.

In Reference 4, for instance, equations were developed for computing discontinuity stresses at cone-cylinder junctures, either with or without transverse reinforcement. For that problem it was tacitly assumed that the shell elements were each of semi-infinite length so that the deformations at their common juncture were not influenced by boundary effects at the others ends. This permitted the use of rather simple expressions for the edge deformations.

For the present problem of the web-stiffened sandwich cylinder, the elements comprising the structure are of such proportions that interaction of internal edge effects is very predominant. This necessitated the development of edge coefficients for cylindrical shell elements of finite length. However, it turns out that the forms of the new coefficients are exactly the same as those of Reference 4 except for multiplying factors which are functions of the shell geometry and, primarily, the length. These edge coefficients for a cylindrical shell are written in the following convenient form:

$$E^s a_i = + \frac{1}{D_i' \beta_i} \Lambda_i^{[2]}(\beta_i l)$$

$$E^s b_i = - \frac{1}{2D_i' \beta_i^2} \Lambda_i^{[1]}(\beta_i l)$$

$$E^s c_i' = E c_i'' = E c_i''' = E f_i' = 0$$

$$E^s d_i = + \frac{1}{2D_i'\beta_i^2} \Lambda_i^{[1]}(\beta_i l)$$

$$E^s f_i'' = + \frac{R_i^2}{h_i}$$

$$E^s f_i''' = - \nu \frac{R_i}{h_i}$$

$$E^s g_i = - \frac{1}{2D_i'\beta_i^3} \Lambda_i^{[3]}(\beta_i l) \qquad [7a]$$

$$E^s a_{ij} = - \frac{1}{D_i'\beta_i} \Lambda_i^{[6]}(\beta_i l)$$

$$E^s b_{ij} = + \frac{1}{2D_i'\beta_i^2} \Lambda_i^{[4]}(\beta_i l)$$

$$E^s c_{ij} = E f_{ij}' = 0$$

$$E^s d_{ij} = - \frac{1}{2D_i'\beta_i^2} \Lambda_i^{[4]}(\beta_i l)$$

$$E^s g_{ij} = + \frac{1}{2D_i'\beta_i^3} \Lambda_i^{[5]}(\beta_i l)$$

where

$$D_i' \equiv \frac{D_i}{E} = \frac{h_i^3}{12(1-\nu^2)} \; ; \; \beta_i = \frac{\sqrt[4]{3(1-\nu^2)}}{\sqrt{R_i h_i}} \qquad [7b]$$

The "lambda" functions $\Lambda_i^{[1]}$, $\Lambda_i^{[2]}$, $\Lambda_i^{[3]}$, . . . $\Lambda_i^{[6]}$ appearing in the edge coefficients, Equations [7a], are derived in Appendix A and are defined here as follows:

$$\Lambda_i^{[1]}(\beta_i l) = \frac{\sinh^2 \beta_i l + \sin^2 \beta_i l}{\sinh^2 \beta_i l - \sin^2 \beta_i l}$$

$$\Lambda_i^{[2]}(\beta_i l) = \frac{\cosh \beta_i l \sinh \beta_i l + \cos \beta_i l \sin \beta_i l}{\sinh^2 \beta_i l - \sin^2 \beta_i l}$$

$$\Lambda_i^{[3]}(\beta_i l) = \frac{\cosh \beta_i l \sinh \beta_i l - \cos \beta_i l \sin \beta_i l}{\sinh^2 \beta_i l - \sin^2 \beta_i l}$$

$$\Lambda_i^{[4]}(\beta_i l) = \frac{2 \sinh \beta_i l \sin \beta_i l}{\sinh^2 \beta_i l - \sin^2 \beta_i l} \qquad [8]$$

$$\Lambda_i^{[5]}(\beta_i l) = \frac{\cosh \beta_i l \sin \beta_i l - \sinh \beta_i l \cos \beta_i l}{\sinh^2 \beta_i l - \sin^2 \beta_i l}$$

$$\Lambda_i^{[6]}(\beta_i l) = \frac{\cosh \beta_i l \sin \beta_i l + \sinh \beta_i l \cos \beta_i l}{\sinh^2 \beta_i l - \sin^2 \beta_i l}$$

For convenience and ease of calculation, numerical values of the "lambda" functions in [8] were determined with the aid of a Burroughs E-101 computer for a range of $\beta_i l$ from 0.40 to 2.50 in increments of 0.02. The results were tabulated and are given in this report as Table 1.

For the special case of a cylindrical shell of semi-infinite length, i.e., $l \to \infty$, the interaction functions given by Equations [8] simplify to

$$\Lambda_i^{[1]} = \Lambda_i^{[2]} = \Lambda_i^{[3]} = 1$$

$$\Lambda_i^{[4]} = \Lambda_i^{[5]} = \Lambda_i^{[6]} = 0 \qquad [9]$$

and the edge coefficients given by Equations [7a] reduce exactly to those given in Reference 4.

From symmetry considerations it is seen that the edges of the web stiffener, which for purposes of analysis is viewed as a circular annulus, do not undergo any rotation. This stems from the fact that a horizontal tangent or zero-slope condition is assumed to exist at the junctures of the webs with the two cylindrical shells. This assumption implies that the edge moments on each shell at the shell-web junctures balance each other, so that there are no net moments to be resisted by the web. Further, it is assumed that the web elements do not take any axial force due to the axial pressure, but that this is all resisted by the cylindrical shells. Thus, the analysis of the web stiffener is reduced to that of a circular annulus subjected to axisymmetric in-plane radial forces on both its inner and outer boundaries;[6] see Figure 2. On the basis of these assumptions, it is necessary to derive edge coefficients for an annulus undergoing radial deflections only. Such coefficients are developed in Appendix B and are given here as follows:

TABLE 1

Numerical Values of the Functions $\Lambda^{[1]}(\beta l)$ through $\Lambda^{[6]}(\beta l)$ for a Range of βl from 0.40 to 2.50 in Increments of 0.02

(βl)	Sin (βl)	Sinh (βl)	Cos (βl)	Cosh (βl)	$\Lambda^{[1]}$	$\Lambda^{[2]}$	$\Lambda^{[3]}$	$\Lambda^{[4]}$	$\Lambda^{[5]}$	$\Lambda^{[6]}$
0.40	0.389418	0.410752	0.921060	1.08107	18.7667	47.0236	5.00122	18.7396	2.49909	46.8236
0.42	0.407760	0.432457	0.913088	1.08950	17.0252	40.6483	4.76331	16.9954	2.37989	40.4383
0.44	0.425939	0.454335	0.904751	1.09837	15.5161	35.3813	4.54707	15.4835	2.27151	35.1613
0.46	0.443948	0.476395	0.896052	1.10767	14.1998	30.9919	4.34968	14.1643	2.17252	30.7619
0.48	0.461779	0.498645	0.886994	1.11742	13.0449	27.3050	4.16877	13.0063	2.08175	27.0650
0.50	0.479425	0.521095	0.877582	1.12762	12.0261	24.1856	4.00237	11.9844	1.99821	23.9357
0.52	0.496880	0.543753	0.867819	1.13827	11.1229	21.5289	3.84882	11.0779	1.92106	21.2690
0.54	0.514135	0.566629	0.857708	1.14937	10.3186	19.2524	3.70670	10.2698	1.84960	18.9826
0.56	0.531186	0.589731	0.847255	1.16094	9.59916	17.2906	3.57477	9.54686	1.78320	17.0108
0.58	0.548023	0.613070	0.836462	1.17296	8.95317	15.5911	3.45199	8.89710	1.72135	15.3013
0.60	0.564642	0.636653	0.825335	1.18546	8.37102	14.1116	3.33744	8.31104	1.66358	13.8118
0.62	0.581035	0.660491	0.813878	1.19843	7.84460	12.8178	3.23034	7.78052	1.60950	12.5080
0.64	0.597195	0.684594	0.802095	1.21188	7.36708	11.6816	3.12998	7.29887	1.55875	11.3619
0.66	0.613116	0.708970	0.789992	1.22582	6.93263	10.6798	3.03577	6.86010	1.51105	10.3502
0.68	0.628793	0.733630	0.777572	1.24024	6.53626	9.79336	2.94715	6.45929	1.46610	9.45377
0.70	0.644217	0.758583	0.764842	1.25516	6.17371	9.00611	2.86366	6.09214	1.42368	8.65657
0.72	0.659384	0.783840	0.751805	1.27059	5.84126	8.30469	2.78487	5.75500	1.38356	7.94523
0.74	0.674287	0.809410	0.738468	1.28652	5.53571	7.67784	2.71040	5.44463	1.34557	7.30845
0.76	0.688921	0.835304	0.724836	1.30297	5.25429	7.11600	2.63992	5.15822	1.30953	6.73670
0.78	0.703279	0.861533	0.710913	1.31993	4.99456	6.61104	2.57311	4.89340	1.27529	6.22184
0.80	0.717356	0.888105	0.696706	1.33743	4.75438	6.15604	2.50972	4.64802	1.24271	5.75694
0.82	0.731145	0.915034	0.682221	1.35546	4.53188	5.74504	2.44949	4.42017	1.21166	5.33607
0.84	0.744643	0.942328	0.667462	1.37403	4.32540	5.37294	2.39220	4.20822	1.18204	4.95409
0.86	0.757842	0.969999	0.652437	1.39316	4.13348	5.03531	2.33765	4.01069	1.15374	4.60661
0.88	0.770738	0.998058	0.637151	1.41284	3.95481	4.72832	2.28565	3.82629	1.12667	4.28978
0.90	0.783326	1.02651	0.621609	1.43308	3.78823	4.44865	2.23604	3.65386	1.10075	4.00028
0.92	0.795601	1.05538	0.605820	1.45390	3.63272	4.19339	2.18867	3.49237	1.07590	3.73521
0.94	0.807558	1.08467	0.589788	1.47530	3.48734	3.95999	2.14339	3.34089	1.05204	3.49202
0.96	0.819191	1.11440	0.573519	1.49729	3.35128	3.74623	2.10008	3.19860	1.02912	3.26848
0.98	0.830497	1.14457	0.557022	1.51988	3.22377	3.55013	2.05863	3.06475	1.00707	3.06262
1.00	0.841470	1.17520	0.540302	1.54308	3.10415	3.36998	2.01891	2.93866	0.985838	2.87273
1.02	0.852108	1.20629	0.523365	1.56689	2.99181	3.20422	1.98084	2.81974	0.965374	2.69726
1.04	0.862404	1.23788	0.506220	1.59133	2.88621	3.05151	1.94433	2.70743	0.945632	2.53486
1.06	0.872355	1.26995	0.488872	1.61641	2.78683	2.91063	1.90928	2.60124	0.926568	2.38431
1.08	0.881957	1.30254	0.471328	1.64213	2.69324	2.78051	1.87562	2.50071	0.908143	2.24455
1.10	0.891207	1.33564	0.453596	1.66851	2.60502	2.66019	1.84328	2.40544	0.890318	2.11461
1.12	0.900100	1.36928	0.435682	1.69556	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

0.92	0.795601	1.05538	0.605820	1.45390	3.63272	4.19339	2.18867	3.49237	1.07590	3.73521
0.94	0.807558	1.08467	0.589788	1.47530	3.48734	3.95999	2.14339	3.34089	1.05204	3.49202
0.96	0.819191	1.11440	0.573519	1.49729	3.35128	3.74623	2.10008	3.19860	1.02912	3.26848
0.98	0.830497	1.14457	0.557022	1.51988	3.22377	3.55013	2.05863	3.06475	1.00707	3.06262
1.00	0.841470	1.17520	0.540302	1.54308	3.10415	3.36998	2.01891	2.93866	0.985838	2.87273
1.02	0.852108	1.20629	0.523365	1.56689	2.99181	3.20422	1.98084	2.81974	0.965374	2.69726
1.04	0.862404	1.23788	0.506220	1.59133	2.88621	3.05151	1.94433	2.70743	0.945632	2.53486
1.06	0.872355	1.26995	0.488872	1.61641	2.78683	2.91063	1.90928	2.60124	0.926568	2.38431
1.08	0.881957	1.30254	0.471328	1.64213	2.69324	2.78051	1.87562	2.50071	0.908143	2.24455
1.10	0.891207	1.33564	0.453596	1.66851	2.60502	2.66019	1.84328	2.40544	0.890318	2.11461
1.12	0.900100	1.36928	0.435682	1.69556	2.52179	2.54879	1.81219	2.31505	0.873061	1.99363
1.14	0.908633	1.40347	0.417594	1.72329	2.44322	2.44556	1.78228	2.22920	0.856338	1.88084
1.16	0.916803	1.43822	0.399339	1.75170	2.36898	2.34980	1.75350	2.14757	0.840120	1.77555
1.18	0.924606	1.47354	0.380924	1.78082	2.29879	2.26008	1.72580	2.06989	0.824379	1.67714
1.20	0.932039	1.50946	0.362357	1.81065	2.23238	2.17825	1.69912	1.99588	0.809090	1.58505
1.22	0.939099	1.54597	1.343645	1.84120	2.16952	2.10138	1.67341	1.92531	0.794227	1.49876
1.24	0.945783	1.58311	0.324796	1.87249	2.10998	2.02982	1.64864	1.85796	0.779769	1.41782
1.26	0.952090	1.62088	0.305816	1.90453	2.05355	1.96317	1.62476	1.79362	0.765694	1.34181
1.28	0.958015	1.65930	0.286715	1.93733	2.00005	1.90102	1.60172	1.73211	0.751982	1.27037
1.30	0.963558	1.69838	0.267498	1.97091	1.94930	1.84305	1.57951	1.67325	0.738[illegible]5	1.20313
1.32	0.968715	1.73814	0.248175	2.00527	1.90113	1.78893	1.55807	1.61688	0.725575	1.13980
1.34	0.973484	1.77859	0.228752	2.04044	1.85539	1.73838	1.53738	1.56285	0.712845	1.08008
1.36	0.977864	1.81976	0.209238	2.07642	1.81196	1.69115	1.51741	1.51102	0.700410	1.02373
1.38	0.981853	1.86166	0.189640	2.11324	1.77069	1.64698	1.49812	1.46128	0.688257	0.970497
1.40	0.985449	1.90430	0.169967	2.15089	1.73146	1.60566	1.47950	1.41349	0.676370	0.920164
1.42	0.988651	1.94770	0.150225	2.18941	1.69417	1.56700	1.46152	1.36755	0.664737	0.872537
1.44	0.991458	1.99188	0.130423	2.22881	1.65870	1.53079	1.44414	1.32336	0.653346	0.827431
1.46	0.993868	2.03686	0.110569	2.26909	1.62496	1.49689	1.42736	1.28083	0.642185	0.784680
1.48	0.955880	2.08265	0.090671	2.31029	1.59287	1.46512	1.41114	1.23985	0.631245	0.744130
1.50	0.997494	2.12927	0.070737	2.35240	1.56232	1.43535	1.39548	1.20036	0.620514	0.705637
1.52	0.998710	2.17675	0.050774	2.39546	1.53325	1.40745	1.38034	1.16227	0.609983	0.669073
1.54	0.999525	2.22510	0.030791	2.43948	1.50558	1.38128	1.36570	1.12552	0.599643	0.634316
1.56	0.999941	2.27434	0.010796	2.48447	1.47924	1.35674	1.35156	1.09002	0.589486	0.601255
1.58	0.999957	2.32449	−0.009203	2.53046	1.45416	1.33372	1.33790	1.05574	0.579503	0.569786
1.60	0.999573	2.37556	−0.029199	2.57746	1.43027	1.31212	1.32469	1.02259	0.569687	0.539815
1.62	0.998789	2.42759	−0.049183	2.62549	1.40753	1.29186	1.31193	0.990537	0.560031	0.511254
1.64	0.997606	2.48059	−0.069148	2.67457	1.38588	1.27285	1.29960	0.959516	0.550529	0.484020
1.66	0.996023	2.53458	−0.089085	2.72472	1.36526	1.25501	1.28768	0.929484	0.541173	0.458039
1.68	0.994043	2.58959	−0.108986	2.77596	1.34562	1.23827	1.27616	0.900394	0.531957	0.433238
1.70	0.991664	2.64563	−0.128844	2.82831	1.32692	1.22256	1.26504	0.872205	0.522877	0.409554
1.72	0.988889	2.70273	−0.148650	2.88179	1.30912	1.20782	1.25429	0.844875	0.513927	0.386924
1.74	0.985719	2.76091	−0.168397	2.93643	1.29217	1.19399	1.24390	0.818369	0.505101	0.365294
1.76	0.982154	2.82019	−0.188076	2.99224	1.27604	1.18101	1.23387	0.792649	0.496396	0.344608
1.78	0.978196	2.88060	−0.207681	3.04924	1.26069	1.16884	1.22419	0.767684	0.487807	0.324819
1.80	0.973847	2.94217	−0.227202	3.10747	1.24607	1.15742	1.21483	0.743441	0.479329	0.305881
1.82	0.969109	3.00491	−0.246632	3.16694	1.23217	1.14672	1.20580	0.719892	0.470958	0.287750
1.84	0.963982	3.06886	−0.265963	3.22767	1.21894	1.13668	1.19709	0.697008	0.462692	0.270387
1.86	0.958471	3.13403	−0.285189	3.28970	1.20636	1.12727	1.18867	0.674764	0.454527	0.253753
1.88	0.952576	3.20045	−0.304300	3.35304	1.19439	1.11846	1.18056	0.653135	0.446459	0.237815
1.90	0.946300	3.26816	−0.323289	3.41773	1.18302	1.11020	1.17272	0.632096	0.438485	0.222539
1.92	0.939645	3.33717	−0.342149	3.48378	1.17221	1.10246	1.16517	0.611628	0.430604	0.207894
1.94	0.932615	3.40752	−0.360872	3.55122	1.16194	1.09522	1.15789	0.591709	0.422811	0.193851
1.96	0.925211	3.47923	−0.379451	3.62009	1.15219	1.08845	1.15087	0.572319	0.415106	0.180384
1.98	0.917437	3.55233	−0.397878	3.69040	1.14293	1.08211	1.14410	0.553440	0.407485	0.167466
2.00	0.909300	3.62686	−0.416140	3.76220	1.13414	1.07619	1.13758	0.535056	0.399946	0.155077
2.02	0.900790	3.70283	−0.434240	3.83549	1.12580	1.07066	1.13130	0.517146	0.392486	0.143187
2.04	0.891930	3.78029	−0.452170	3.91032	1.11790	1.06548	1.12526	0.499701	0.385108	0.131781

1.48	0.955880	2.08265	0.090671	2.31029	1.59287	1.46512	1.41114	1.23985	0.631245	0.744130
1.50	0.997494	2.12927	0.070737	2.35240	1.56232	1.43535	1.39548	1.20036	0.620514	0.705637
1.52	0.998710	2.17675	0.050774	2.39546	1.53325	1.40745	1.38034	1.16227	0.609983	0.669073
1.54	0.999525	2.22510	0.030791	2.43948	1.50558	1.38128	1.36570	1.12552	0.599643	0.634316
1.56	0.999941	2.27434	0.010796	2.48447	1.47924	1.35674	1.35156	1.09002	0.589486	0.601255
1.58	0.999957	2.32449	−0.009203	2.53046	1.45416	1.33372	1.33790	1.05574	0.579503	0.569786
1.60	0.999573	2.37556	−0.029199	2.57746	1.43027	1.31212	1.32469	1.02259	0.569687	0.539815
1.62	0.998789	2.42759	−0.049183	2.62549	1.40753	1.29186	1.31193	0.990537	0.560031	0.511254
1.64	0.997606	2.48059	−0.069148	2.67457	1.38588	1.27285	1.29960	0.959516	0.550529	0.484020
1.66	0.996023	2.53458	−0.089085	2.72472	1.36526	1.25501	1.28768	0.929484	0.541173	0.458039
1.68	0.994043	2.58959	−0.108986	2.77596	1.34562	1.23827	1.27616	0.900394	0.531957	0.433238
1.70	0.991664	2.64563	−0.128844	2.82831	1.32692	1.22256	1.26504	0.872205	0.522877	0.409554
1.72	0.988889	2.70273	−0.148650	2.88179	1.30912	1.20782	1.25429	0.844875	0.513927	0.386924
1.74	0.985719	2.76091	−0.168397	2.93643	1.29217	1.19399	1.24390	0.818369	0.505101	0.365294
1.76	0.982154	2.82019	−0.188076	2.99224	1.27604	1.18101	1.23387	0.792649	0.496396	0.344608
1.78	0.978196	2.88060	−0.207681	3.04924	1.26069	1.16884	1.22419	0.767684	0.487807	0.324819
1.80	0.973847	2.94217	−0.227202	3.10747	1.24607	1.15742	1.21483	0.743441	0.479329	0.305881
1.82	0.969109	3.00491	−0.246632	3.16694	1.23217	1.14672	1.20580	0.719892	0.470958	0.287750
1.84	0.963982	3.06886	−0.265963	3.22767	1.21894	1.13668	1.19709	0.697008	0.462692	0.270387
1.86	0.958471	3.13403	−0.285189	3.28970	1.20636	1.12727	1.18867	0.674764	0.454527	0.253753
1.88	0.952576	3.20045	−0.304300	3.35304	1.19439	1.11846	1.18056	0.653135	0.446459	0.237815
1.90	0.946300	3.26816	−0.323289	3.41773	1.18302	1.11020	1.17272	0.632096	0.438485	0.222539
1.92	0.939645	3.33717	−0.342149	3.48378	1.17221	1.10246	1.16517	0.611628	0.430604	0.207894
1.94	0.932615	3.40752	−0.360872	3.55122	1.16194	1.09522	1.15789	0.591709	0.422811	0.193851
1.96	0.925211	3.47923	−0.379451	3.62009	1.15219	1.08845	1.15087	0.572319	0.415106	0.180384
1.98	0.917437	3.55233	−0.397878	3.69040	1.14293	1.08211	1.14410	0.553440	0.407485	0.167466
2.00	0.909300	3.62686	−0.416140	3.76220	1.13414	1.07619	1.13758	0.535056	0.399946	0.155077
2.02	0.900790	3.70283	−0.434240	3.83549	1.12580	1.07066	1.13130	0.517146	0.392486	0.143187
2.04	0.891930	3.78029	−0.452170	3.91032	1.11790	1.06548	1.12526	0.499701	0.385108	0.131781
2.06	0.882710	3.85926	−0.469920	3.98671	1.11040	1.06066	1.11943	0.482702	0.377808	0.120836
2.08	0.873130	3.93977	−0.487480	4.06470	1.10330	1.05616	1.11383	0.466133	0.370581	0.110333
2.10	0.863210	4.02186	−0.504840	4.14431	1.09658	1.05196	1.10845	0.449987	0.363429	0.100258
2.12	0.852940	4.10555	−0.522000	4.22558	1.09021	1.04805	1.10326	0.434248	0.356352	0.090591
2.14	0.842330	4.19089	−0.538960	4.30855	1.08419	1.04441	1.09829	0.418903	0.349349	0.081315
2.16	0.831380	4.27791	−0.555700	4.39323	1.07850	1.04103	1.09350	0.403941	0.342413	0.072416
2.18	0.820106	4.36663	−0.572210	4.47967	1.07312	1.03788	1.08890	0.389357	0.335551	0.063885
2.20	0.808500	4.45711	−0.588500	4.56791	1.06804	1.03496	1.08449	0.375134	0.328758	0.055701
2.22	0.796570	4.54936	−0.604550	4.65797	1.06325	1.03225	1.08026	0.361265	0.322034	0.047855
2.24	0.784320	4.64344	−0.620360	4.74989	1.05873	1.02973	1.07619	0.347739	0.315378	0.040332
2.26	0.771750	4.73937	−0.635920	4.84372	1.05447	1.02741	1.07230	0.334546	0.308789	0.033123
2.28	0.758880	4.83720	−0.651230	4.93948	1.05046	1.02525	1.06856	0.321685	0.302270	0.026217
2.30	0.745710	4.93696	−0.666270	5.03722	1.04669	1.02326	1.06499	0.309145	0.295818	0.019605
2.32	0.732230	5.03870	−0.681050	5.13697	1.04314	1.02143	1.06156	0.296912	0.289431	0.013271
2.34	0.718470	5.14245	−0.695560	5.23878	1.03981	1.01974	1.05828	0.284990	0.283115	0.007213
2.36	0.704410	5.24827	−0.709790	5.34269	1.03668	1.01818	1.05515	0.273359	0.276862	0.001415
2.38	0.690080	5.35618	−0.723730	5.44873	1.03375	1.01674	1.05215	0.262025	0.270677	−0.004124
2.40	0.675470	5.46623	−0.737390	5.55695	1.03101	1.01543	1.04928	0.250974	0.264561	−0.009420
2.42	0.660580	5.57847	−0.750750	5.66739	1.02844	1.01422	1.04655	0.240200	0.258508	−0.014479
2.44	0.645440	5.69294	−0.763810	5.78010	1.02604	1.01312	1.04394	0.229703	0.252525	−0.019304
2.46	0.630030	5.80969	−0.776570	5.89512	1.02380	1.01211	1.04144	0.219470	0.246607	−0.023909
2.48	0.614380	5.92876	−0.789010	6.01250	1.02171	1.01119	1.03907	0.209503	0.240757	−0.028294
2.50	0.598470	6.05020	−0.801140	6.13229	1.01976	1.01035	1.03681	0.199789	0.234974	−0.032473

3

$$E^A g_1^A = + \mathcal{B}\left[\frac{1}{\bar{R}_1} + \left(\frac{1-\nu}{1+\nu}\right)\frac{\bar{R}_1}{\bar{R}_2^2}\right]$$

$$E^A g_{12}^A = + \mathcal{B}\left[\left(\frac{2}{1+\nu}\right)\frac{1}{\bar{R}_1}\right]$$

$$E^A g_2^A = + \mathcal{B}\left[\frac{1}{\bar{R}_2} + \left(\frac{1-\nu}{1+\nu}\right)\frac{\bar{R}_2}{\bar{R}_1^2}\right] \qquad [10a]$$

$$E^A g_{21}^A = + \mathcal{B}\left[\left(\frac{2}{1+\nu}\right)\frac{1}{\bar{R}_2}\right]$$

where

$$\bar{R}_1 = R_1 + \frac{h_1}{2}\ ; \ \bar{R}_2 = R_2 - \frac{h_2}{2}$$

$$\mathcal{B} = \frac{(1+\nu)\,\bar{R}_1^2\,\bar{R}_2^2}{h_3\,(\bar{R}_1^2 - \bar{R}_2^2)} \qquad [10b]$$

COMPUTATION OF STRESSES

The formulas given herein for determining the longitudinal and circumferential stresses in the shell elements of the web-stiffened sandwich cylinder are developed in Appendix C. Formulas for the radial and tangential stresses in the web elements are developed in Appendix B. The derivation follows very closely the general analysis of Reference 7 for ring-stiffened cylinders under hydrostatic pressure, the only differences arising from the elastic restraints at the shell edges and the distribution of the axial pressure loading.

The nomenclature and sign convention used in Reference 7 and in Appendix C of this report are shown in Figures 2 and 4. A longitudinal bending moment M_x is considered positive if it tends to put the outer surface of the shell in tension, and a transverse shearing force Q_x is considered positive when it acts in a direction away from the axis of symmetry but in the positive x-direction. A hydrostatic pressure p is considered positive when it is external, and negative when internal. With reference to Equations [1] and [2], the subscript i is used to distinguish the two cylinder elements.

The quantities H_i and M_i shown in Figure 2 are the edge shearing forces and bending moments arising at the junctures of the shell elements with a web stiffener. They may be determined in terms of the geometry and elasticity of the structure and the pressure loading by enforcing conditions of force and moment equilibrium and compatibility of deformations at the junctures. This determination is developed in the next section.

Once the edge forces and moments are known, the following formulas may be used for determining the critical longitudinal and circumferential shell stresses which occur at a point midbay between two adjacent webs and also at a web location, respectively:

AT MIDBAY:

$$\sigma_{X_m} = -\frac{P_i}{h_i} \mp \frac{E^s h_i \beta_i^2}{(1-\nu^2)} (w_i^p - w_i^s) \frac{\Lambda_i^{[5]}(\beta_i l/2)}{\Lambda_i^{[2]}(\beta_i l/2)} \qquad [11]$$

$$\sigma_{\Phi_m} = -\nu \frac{P_i}{h_i} + \frac{E^s}{R_i} (-w_i^s) \frac{\Lambda_i^{[6]}(\beta_i l/2)}{\Lambda_i^{[2]}(\beta_i l/2)} \mp \frac{\nu E h_i \beta_i^2}{(1-\nu^2)} (w_i^p - w_i^s) \frac{\Lambda_i^{[5]}(\beta_i l/2)}{\Lambda_i^{[2]}(\beta_i l/2)} \qquad [12]$$

AT A WEB:

$$\sigma_{X_f} = -\frac{P_i}{h_i} \pm \frac{E^s h_i \beta_i^2}{(1-\nu^2)} (w_i^p - w_i^s) \frac{\Lambda_i^{[3]}(\beta_i l/2)}{\Lambda_i^{[2]}(\beta_i l/2)} \qquad [13]$$

$$\sigma_{\Phi_f} = -\nu \frac{P_i}{h_i} + \frac{E^s}{R_i} (-w_i^s) \pm \frac{\nu E^s h_i \beta_i^2}{(1-\nu^2)} (w_i^p - w_i^s) \frac{\Lambda_i^{[3]}(\beta_i l/2)}{\Lambda_i^{[2]}(\beta_i l/2)} \qquad [14]$$

where in the above equations $i = 1, 2$, and the upper sign is for the outer fiber and the lower sign for the inner fiber of each shell plating. Equations [11] through [14] are developed in Appendix C.

Once the critical stresses are determined from Equations [11] through [14], the question as to how they combine to precipitate axisymmetric collapse of the cylindrical shell elements can be answered by recourse to the failure criteria discussed in References 7 and 8. This will not be discussed here.

The quantity P_i in Equations [11] through [14] is the axial load taken by each of the two cylindrical shells. On the assumption that the two shells contract the same amount longitudinally, it is shown in Appendix D that

$$P_1 = \frac{pR_1\left[\nu(1-\alpha_1)+\frac{1}{2}\frac{R_1h_1}{R_2h_2}(1-\nu^2\alpha_2)\right]+\frac{\nu E^s h_1}{R_1}(g_1^A H_1+\bar{g}_1^A H_2)\alpha_1-\frac{\nu E^s h_1}{R_2}(g_2^A H_2+\bar{g}_2^A H_1)\alpha_2}{1-\nu^2\alpha_1+\frac{R_1h_1}{R_2h_2}(1-\nu^2\alpha_2)}$$

[15]

$$P_2 = \frac{pR_1\left[-\nu(1-\alpha_1)\frac{R_1}{R_2}+\frac{1}{2}\frac{R_1}{R_2}(1-\nu^2\alpha_1)\right]-\frac{\nu E^s h_1}{R_2}(g_1^A H_1+\bar{g}_1^A H_2)\alpha_1+\frac{\nu E^s R_1 h_1}{R_2^2}(g_2^A H_2+\bar{g}_2^A H_1)\alpha_2}{1-\nu^2\alpha_1+\frac{R_1h_1}{R_2h_2}(1-\nu^2\alpha_2)}$$

The quantities α_1 and α_2 are given in Appendix D by Equations [D.13].

The quantities w_i^p in Equations [11] through [14] represent the particular integrals to the differential equations governing the axisymmetric deformations of a cylindrical shell; they are easily determined from membrane theory. For the case shown in Figures 1 and 2, where the outer cylindrical shell (but not the inner one) is loaded by lateral pressure, we find that (see Reference 7, for example)

$$w_1^p = +\frac{pR_1^2}{E^s h_1}\left(1-\frac{\nu}{R_1}\frac{P_1}{p}\right)$$

[16]

$$w_2^p = -\frac{pR_2^2}{E^s h_2}\left(\frac{\nu}{R_2}\frac{P_2}{p}\right)$$

where the axial forces P_1 and P_2 are given by Equations [15].

The shell edge deflections w_i^s appearing in the stress formulas, Equations [11] through [14], are determined from Equation [3] once the edge shears H_i and edge moments M_i are known; i.e.,

$$w_1^s = d_1M_1 + g_1\frac{H_1}{2} + f_1'P_1 + f_1''p + f_1'''P_1 + \bar{d}_1M_1 + \bar{g}_1\frac{H_1}{2} + \bar{f}_1'P_1$$

[17]

$$w_2 = d_2M_2 + g_2\frac{H_2}{2} + f_2'P_2 + f_2''p + f_2'''P_2 + \bar{d}_2M_2 + \bar{g}_2\frac{H_2}{2} + \bar{f}_2'P_2$$

where the interaction coefficients have been designated by a "bar" instead of the double subscript so as not to confuse the use of the subscripts "1" and "2" to designate the two shells and their respective junctures with the web stiffeners. This notation will be used in all the equations that follow.

Expressions for the radial and tangential stresses in the web elements are developed in Appendix B. It is shown there that the maximum radial stress occurs at the intersection with the outer cylindrical shell $\left(\text{i.e., at } r = \bar{R}_1 = R_1 + \frac{h_1}{2}\right)$, and the maximum tangential stress occurs at the intersection with the inner cylindrical shell $\left(\text{i.e., at } r = \bar{R}_2 = R_2 - \frac{h_2}{2}\right)$. These maximum stresses are given by the following expressions:

$$\sigma_{r_{\max}} = \frac{E^A}{(1-\nu^2)} \left[A(1+\nu) - \frac{B}{\bar{R}_1^2} (1-\nu) \right] \qquad [18]$$

$$\sigma_{t_{\max}} = \frac{E^A}{(1-\nu^2)} \left[A(1+\nu) + \frac{B}{\bar{R}_2^2} (1-\nu) \right] \qquad [19]$$

where the constants A and B are given by

$$A = -\left(\frac{w_1^A \bar{R}_1 - w_2^A \bar{R}_2}{\bar{R}_1^2 - \bar{R}_2^2} \right)$$

$$B = -\bar{R}_1 \bar{R}_2 \left(\frac{w_2^A \bar{R}_1 - w_1^A \bar{R}_2}{\bar{R}_1^2 - \bar{R}_2^2} \right) \qquad [20]$$

and the annulus edge deflections w_1^A and w_2^A by Equation [5] as

$$w_1^A = g_1^A H_1 + \bar{g}_1^A H_2$$

$$w_2^A = g_2^A H_2 + \bar{g}_2^A H_1 \qquad [21]$$

In Equations [21] the edge coefficients designated by a "bar" are the interaction or double-subscript coefficients; i.e., $\bar{g}_1^A \equiv g_{12}^A$ and $\bar{g}_2^A \equiv g_{21}^A$. The edge coefficients appearing in Equations [21] are given by Equations [10].

DETERMINATION OF EDGE SHEARS H_i AND EDGE MOMENTS M_i

For the case of symmetry on each side of a web stiffener, the conditions of force and moment equilibrium at each of the two junctures of the web with the shells are rather obvious; these are shown in the free-body diagram of Figure 2. There remains to determine the unknown edge shears H_1 and H_2 and unknown edge moments M_1 and M_2 by enforcing conditions of compatibility of the deformations at the junctures labeled "1" and "2."

Continuity and symmetry conditions at joint "1" require that

$$w_1^s = w_1^A \qquad [22]$$

$$\theta_1^s = \theta_1^A = 0 \qquad [23]$$

whereas these conditions applied to joint "2" require that

$$w_2^s = w_2^A \qquad [24]$$

$$\theta_2^s = \theta_2^A = 0 \qquad [25]$$

Substituting Equations [3], [4], [5], and [6] into the four conditions [22] through [25], considering the zero edge coefficients by virtue of the loading shown in Figures 1 and 2 and Equations [7], and assuming that $E^s = E^A$, we must solve the following four algebraic equations simultaneously to determine H_1, H_2, M_1, and M_2:

$$M_1\,[d_1+\bar{d}_1] + H_1\left[\frac{1}{2}\,(g_1+\bar{g}_1) - g_1^A\right] + H_2\,[-\bar{g}_1^A] = -f_1''p - f_1'''P_1 \qquad [26]$$

$$M_1\,[a_1+\bar{a}_1] + H_1\left[\frac{1}{2}\,(b_1+\bar{b}_1)\right] = 0 \qquad [27]$$

$$M_2\,[d_2+\bar{d}_2] + H_2\left[\frac{1}{2}\,(g_2+\bar{g}_2) - g_2^A\right] + H_1\,[-\bar{g}_2^A] = -f_2'''P_2 \qquad [28]$$

$$M_2\,[a_2+\bar{a}_2] + H_2\left[\frac{1}{2}\,(b_2+\bar{b}_2)\right] = 0 \qquad [29]$$

where P_1 and P_2 are given by Equations [15] to be functions of the unknown shearing forces H_1 and H_2. Equations [26] through [29] can be rewritten as two equations in only two unknowns as follows:

$$H_1\left[\frac{1}{2}\,(g_1+\bar{g}_1) - g_1^A - \frac{(b_1+\bar{b}_1)\,(d_1+\bar{d}_1)}{2\,(a_1+\bar{a}_1)}\right] + H_2\,[-\bar{g}_1^A] = -f_1''p - f_1'''P_1 \qquad [30]$$

$$H_1\,[-\bar{g}_2^A] + H_2\left[\frac{1}{2}\,(g_2+\bar{g}_2) - g_2^A - \frac{(b_2+\bar{b}_2)\,(d_2+\bar{d}_2)}{2\,(a_2+\bar{a}_2)}\right] = -f_2'''P_2 \qquad [31]$$

By the substitution of the expressions for P_1 and P_2 given by Equations [15] into Equations [30] and [31], the two simultaneous equations to be solved for H_1 and H_2 become:

$$H_1\left[\frac{1}{2}(g_1+\bar{g}_1)-g_1^A-\frac{(b_1+\bar{b}_1)(d_1+\bar{d}_1)}{2(a_1+\bar{a}_1)}+\frac{\nu E^s h_1}{R_1}f_1'''\left(\frac{\mathcal{a}_1 g_1^A-\frac{R_1}{R_2}\mathcal{a}_2\bar{g}_2^A}{1-\nu^2\mathcal{a}_1+\frac{R_1h_1}{R_2h_2}(1-\nu^2\mathcal{a}_2)}\right)\right]+$$

$$H_2\left[-\bar{g}_1^A+\frac{\nu E^s h_1}{R_1}f_1'''\left(\frac{\mathcal{a}_1\bar{g}_1^A-\frac{R_1}{R_2}\mathcal{a}_2 g_2^A}{1-\nu^2\mathcal{a}_1+\frac{R_1h_1}{R_2h_2}(1-\nu^2\mathcal{a}_2)}\right)\right]=-f_1''p-f_1'''\frac{pR_1\left[\nu(1-\mathcal{a}_1)+\frac{1}{2}\frac{R_1h_1}{R_2h_2}(1-\nu^2\mathcal{a}_2)\right]}{1-\nu^2\mathcal{a}_1+\frac{R_1h_1}{R_2h_2}(1-\nu^2\mathcal{a}_2)} \qquad [32]$$

$$H_1\left[-\bar{g}_2^A+\frac{\nu E^s h_1}{R_2}f_2'''\left(\frac{-\mathcal{a}_1 g_1^A+\frac{R_1}{R_2}\mathcal{a}_2\bar{g}_2^A}{1-\nu^2\mathcal{a}_1+\frac{R_1h_1}{R_2h_2}(1-\nu^2\mathcal{a}_2)}\right)\right]+$$

$$H_2\left[\frac{1}{2}(g_2+\bar{g}_2)-g_2^A-\frac{(b_2+\bar{b}_2)(d_2+\bar{d}_2)}{2(a_2+\bar{a}_2)}+\frac{\nu E^s h_1}{R_2}f_2'''\left(\frac{-\mathcal{a}_1\bar{g}_1^A+\frac{R_1}{R_2}\mathcal{a}_2 g_2^A}{1-\nu^2\mathcal{a}_1+\frac{R_1h_1}{R_2h_2}(1-\nu^2\mathcal{a}_2)}\right)\right]$$

$$=-f_2'''\frac{pR_1\left[-\nu(1-\mathcal{a}_1)\frac{R_1}{R_2}+\frac{1}{2}\frac{R_1}{R_2}(1-\nu^2\mathcal{a}_1)\right]}{1-\nu^2\mathcal{a}_1+\frac{R_1h_1}{R_2h_2}(1-\nu^2\mathcal{a}_2)} \qquad [33]$$

After the edge shears H_1 and H_2 are determined from Equations [32] and [33], the edge moments M_1 and M_2 may be found from the following expressions as a consequence of Equations [27] and [29], respectively:

$$M_1=-H_1\frac{(b_1+\bar{b}_1)}{2(a_1+\bar{a}_1)} \qquad [34]$$

$$M_2=-H_2\frac{(b_2+\bar{b}_2)}{2(a_2+\bar{a}_2)} \qquad [35]$$

NUMERICAL EXAMPLE

As part of its research and evaluation program to study the application of glass-fiber reinforced plastics for pressure vessel construction, the Model Basin in collaboration with Narmco Industries Inc., San Diego, California, is presently designing a series of web-stiffened sandwich cylinders made of these materials. The structural models are to be fabricated by Narmco and then forwarded to the Model Basin for testing.

One of the designs, Model N-1, presently being conceived will be used as a sample calculation to illustrate the use of the equations developed in this report. The detailed dimensions are shown in Figure 6, and are summarized here:

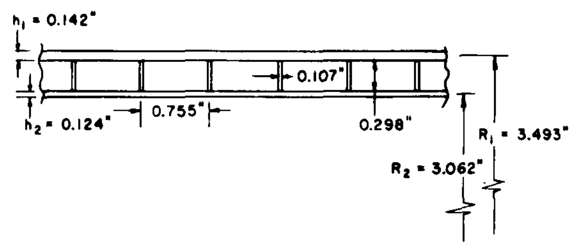


Figure 6 – Schematic Diagram Showing Dimensions of Model N-1

$$h_1 = 0.142 \text{ in.}; \quad h_2 = 0.124 \text{ in.}; \quad h_3 = 0.107 \text{ in.}$$
$$R_1 = 3.493 \text{ in.}; \quad R_2 = 3.062 \text{ in.}; \quad l = 0.648 \text{ in.}$$
$$\bar{R}_1 = 3.564 \text{ in.}; \quad \bar{R}_2 = 3.000 \text{ in.}$$
$$E^s = 6.0 \times 10^6 \text{ psi} = E^A$$
$$\nu = 0.15$$

Using Equations [7b] for each of the outer and inner cylindrical shells, respectively, we compute the values of $\beta_i l$ and D_i' to be:

$$\beta_1 l = 1.204; \quad D_1' = 24.407 \times 10^{-5} \text{ in.}^3$$
$$\beta_2 l = 1.376; \quad D_2' = 16.257 \times 10^{-5} \text{ in.}^3$$

The lambda functions are either computed by using these values of $\beta_i l$ and Equations [8] or are found by interpolation from Table 1 for each of the two shells. They are summarized here:

	$\Lambda^{[1]}$	$\Lambda^{[2]}$	$\Lambda^{[3]}$	$\Lambda^{[4]}$	$\Lambda^{[5]}$	$\Lambda^{[6]}$
Outer Shell	2.219	2.162	1.694	1.981	0.8061	1.567
Inner Shell	1.778	1.655	1.502	1.471	0.6906	0.9805

Next, the shell-edge coefficients $E^s a_i$, $E^s b_i$, $E^s c_i'$, . . . etc., are computed by using Equations [7a]. The numerical values thus found are summarized here:

Shell Edge Coefficient	Outer Shell ($i = 1$)	Inner Shell ($i = 2$)
$E^s a_i$, in.$^{-2}$	$+ 0.4767 \times 10^4$	$+0.4795 \times 10^4$
$E^s b_i$, in.$^{-1}$	$- 0.1317 \times 10^4$	-0.1213×10^4
$E^s c_i' = E^s \bar{c}_i'$	0	0
$E^s d_i$, in.$^{-1}$	$+ 0.1317 \times 10^4$	$+0.1213 \times 10^4$
$E^s f_i'$	0	0
$E^s f_i''$, in.	$+85.9229$	nonexistent
$E^s f_i'''$	$- 3.6898$	-3.7040
$E^s g_i$	$- 0.05410 \times 10^4$	-0.04822×10^4
$E^s \bar{a}_i$, in.$^{-2}$	$- 0.3455 \times 10^4$	-0.2840×10^4
$E^s \bar{b}_i$, in.$^{-1}$	$+ 0.1176 \times 10^4$	$+0.1003 \times 10^4$
$E^s \bar{d}_i$, in.$^{-1}$	$- 0.1176 \times 10^4$	-0.1003×10^4
$E^s \bar{f}_i'$	0	0
$E^s \bar{g}_i$	$+ 0.02575 \times 10^4$	$+0.02217 \times 10^4$

The web stiffener or circular annulus edge coefficients $E^A g_1^A$, . . . etc., are computed by using Equations [10], and the numerical results found are:

$$E^A g_1^A = 190.261$$

$$E^A \bar{g}_1^A = E^A g_{12}^A = 161.949$$

$$E^A g_2^A = 168.564$$

$$E^A \bar{g}_2^A = E^A g_{21}^A = 192.396$$

The components of the end pressure loading taken by each of the outer and inner cylindrical shells, respectively, are computed to be, using Equations [15]:

$$P_1 = + 0.9843\, p \text{ lb/in.}$$

$$P_2 = + 0.8695\, p \text{ lb/in.}$$

With all this, the edge shear forces H_1 and H_2 are computed by solving Equations [32] and [33] simultaneously. The values thus found are then substituted into Equations [34] and [35] to determine the edge bending moments M_1 and M_2. The numerical values thus found are:

$$H_1 = + 0.3893\, p \text{ lb/in.}$$

$$H_2 = - 0.2717\, p \text{ lb/in.}$$

$$M_1 = +\ 0.02095\, p \text{ in.-lb/in.}$$
$$M_2 = -\ 0.01459\, p \text{ in.-lb/in.}$$

When the edge shear forces and edge bending moments are known, the edge deflections of the two cylindrical shells and those of the web stiffener, at their common juncture points, are found from Equations [17] and [21] to be:

$$E^s w_1^s = +\ 30.072\, p \text{ lb/in.}$$
$$E^A w_1^A = +\ 30.072\, p \text{ lb/in.}$$
$$E^s w_2^s = +\ 29.105\, p \text{ lb/in.}$$
$$E^A w_2^A = +\ 29.105\, p \text{ lb/in.}$$

Comparison of $E^s w_1^s$ with $E^A w_1^A$, and $E^s w_2^s$ with $E^A w_2^A$ affords a check on the numerical calculations, since the boundary conditions [22] and [24] enforced at the two junctures require them to be equal in their respective cases since it was assumed that $E^s = E^A$.

The maximum radial and tangential stresses in the web stiffeners can now be computed by using Equations [18], [19], and [20]. The values found are:

$$\sigma_{r_{max}} = -\ \ 3.638\, p \text{ lb/in.}^2$$
$$\sigma_{t_{max}} = -\ 10.083\, p \text{ lb/in.}^2$$

Before the shell stresses can be computed, it is necessary to determine the membrane deflections of the two shells. This is done with the aid of Equations [16]. The values found are:

$$E^s w_1^p = +\ 82.291\ \ p \text{ lb/in.}$$
$$E^s w_2^p = -\ \ 3.2205\, p \text{ lb/in.}$$

Finally, the critical longitudinal and circumferential shell stresses at points midbay between two adjacent web stiffeners and at a web stiffener are determined by using Equations [11], [12], [13], and [14]. The numerical values are summarized as follows:

	σ_{X_m}, psi	σ_{Φ_m}, psi	σ_{X_f}, psi	σ_{Φ_f}, psi
Outer Shell	$-10.039\, p$	$-10.437\, p$	$-13.166\, p$	$-10.584\, p$
Inner Shell	$-\ 9.843\, p$	$-12.702\, p$	$-12.704\, p$	$-13.514\, p$

For the numerical example considered, the calculations already carried out have been based on the assumption that all structural elements have the same elastic modulus E. However, in the fabrication of a shell structure such as this, it is conceivable that the elements could have different material properties. In the case of Model N-1, which is to be made of a glass-fiber reinforced plastic, it is expected that the web stiffeners, although made of the same basic material as the cylindrical shells, will have a higher elastic modulus by virtue of the fiber distribution. Assuming that the modulus of the web material is 50 percent higher than that of the shell material, i.e., $E^A = 1.5\, E^s$, we repeated the calculations and found the following results:

$$P_1 = +\,0.9842\,p \text{ lb/in.}$$
$$P_2 = +\,0.8696\,p \text{ lb/in.}$$

$$H_1 = +\,0.4166\,p \text{ lb/in.}$$
$$H_2 = -\,0.2449\,p \text{ lb/in.}$$
$$M_1 = +\,0.02242\,p \text{ in.-lb/in.}$$
$$M_2 = -\,0.01315\,p \text{ in.-lb/in.}$$

$$E^s w_1^s = +\,26.406\ p \text{ lb/in.}$$
$$E^A w_1^A = +\,39.609\ p \text{ lb/in.}$$
$$E^s w_2^s = +\,25.919\ p \text{ lb/in.}$$
$$E^A w_2^A = +\,38.878\ p \text{ lb/in.}$$
$$E^s w_1^P = +\,82.291\ p \text{ lb/in.}$$
$$E^s w_2^P = -\ 3.2211\,p \text{ lb/in.}$$

$$\sigma_{r_{max}} = -\ 3.894\,p \text{ lb/in.}^2$$
$$\sigma_{t_{max}} = -\,13.303\,p \text{ lb/in.}^2$$

	$\sigma_{\chi m}$, psi	$\sigma_{\Phi m}$, psi	$\sigma_{\chi f}$, psi	$\sigma_{\Phi f}$, psi
Outer Shell	$-10.256\,p$	$-\ 9.442\,p$	$-13.603\,p$	$-\ 9.600\,p$
Inner Shell	$-\ 9.565\,p$	$-11.658\,p$	$-12.144\,p$	$-12.390\,p$

ACKNOWLEDGMENTS

Mr. William E. Ball, Jr. checked all the equations in this report and also carried out the calculations for Table 1 and the numerical example. Mr. Ball has also programmed the pertinent equations of the analysis for the Model Basin IBM 7090 computer. This will permit optimization in the design of web-stiffened sandwich cylinders for prescribed material properties and weight-volume ratios.

APPENDIX A

DERIVATION OF THE FUNCTIONS $\Lambda^{[1]}$, $\Lambda^{[2]}$, $\Lambda^{[3]}$, $\Lambda^{[4]}$, $\Lambda^{[5]}$, AND $\Lambda^{[6]}$

If the beam-column effect[7] due to the axial portion of the hydrostatic pressure is neglected, then the differential equation governing the axisymmetric elastic deformations, based on small-deflection theory, of a thin-walled circular cylinder is given by:[7]

$$D \frac{d^4 w}{dx^4} + E \frac{h}{R^2} w = P_r - \frac{\nu}{R} N_x \qquad \text{[A.1]}$$

The homogeneous form of Equation [A.1] will be used to derive edge coefficients for cylindrical shells of short length in which interaction effects between the two ends of the shell prevail. Then we have

$$D \frac{d^4 w}{dx^4} + E \frac{h}{R^2} w = 0 \qquad \text{[A.2]}$$

The solution[7] of Equation [A.2], which solution describes the bending deformations, can be written in the form:

$$\begin{aligned} w_b(x) = {} & C_1 \cos\beta x \cdot \cosh\beta x + C_2 \sin\beta x \cdot \cosh\beta x \\ & + C_3 \cos\beta x \cdot \sinh\beta x + C_4 \sin\beta x \cdot \sinh\beta x \end{aligned} \qquad \text{[A.3]}$$

and the first three derivatives of [A.3] are:

$$\begin{aligned} \frac{1}{\beta} \cdot \frac{dw_b}{dx} = {} & (C_2 + C_3) \cos\beta x \cdot \cosh\beta x + (C_4 - C_1) \sin\beta x \cdot \cosh\beta x \\ & + (C_4 + C_1) \cos\beta x \cdot \sinh\beta x + (C_2 - C_3) \sin\beta x \cdot \sinh\beta x \\[2ex] \frac{1}{2\beta^2} \cdot \frac{d^2 w_b}{dx^2} = {} & C_4 \cos\beta x \cdot \cosh\beta x - C_3 \sin\beta x \cdot \cosh\beta x \\ & + C_2 \cos\beta x \cdot \sinh\beta x - C_1 \sin\beta x \cdot \sinh\beta x \\[2ex] \frac{1}{2\beta^3} \cdot \frac{d^3 w_b}{dx^3} = {} & (C_2 - C_3) \cos\beta x \cdot \cosh\beta x - (C_4 + C_1) \sin\beta x \cdot \cosh\beta x \\ & + (C_4 - C_1) \cos\beta x \cdot \sinh\beta x - (C_2 + C_3) \sin\beta x \cdot \sinh\beta x \end{aligned} \qquad \text{[A.4]}$$

where in Equations [A.3] and [A.4] we have $\beta = \dfrac{\sqrt[4]{3(1-\nu^2)}}{\sqrt{Rh}}$.

The integration constants C_1, C_2, C_3, and C_4 appearing in Equations [A.3] and [A.4] will be determined from a consideration of the load boundary conditions at the edges of the shell element; see Figure 4. The longitudinal bending moment M_x and the transverse shearing force Q_x are related to the derivatives of $w_b(x)$ by the following equations:

$$M_x = + D \frac{d^2 w_b}{dx^2}$$

$$Q_x = \frac{dM_x}{dx} = + D \frac{d^3 w_b}{dx^3} \qquad \text{[A.5]}$$

With reference to Figures 4 and 7, let it be prescribed that the load boundary conditions are given by:

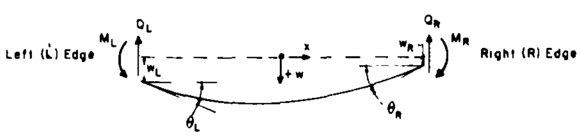


Figure 7 – Edge Shears, Moments, Deflections, and Rotations For a Cylindrical-Shell Element

$$\text{at } x = + \frac{l}{2} : \; M_x = M_R; \quad Q_x = Q_R \qquad \text{[A.6]}$$

$$\text{at } x = - \frac{l}{2} : \; M_x = M_L; \; -Q_x = Q_L \qquad \text{[A.7]}$$

No considerations of symmetry with respect to the point $x=0$ have been taken in writing the solution Equation [A.3], and in formulating the boundary conditions, Equations [A.6] and [A.7]. The development to follow will be general in this sense.

The substitution of Equations [A.4] and [A.5] into the boundary conditions, Equations [A.6] and [A.7], results in the following four equations:

$$\begin{aligned} \frac{M_R}{2D\beta^2} &= C_4 \cos \frac{\beta l}{2} \cdot \cosh \frac{\beta l}{2} - C_3 \sin \frac{\beta l}{2} \cdot \cosh \frac{\beta l}{2} \\ &\quad + C_2 \cos \frac{\beta l}{2} \cdot \sinh \frac{\beta l}{2} - C_1 \sin \frac{\beta l}{2} \cdot \sinh \frac{\beta l}{2} \end{aligned}$$

$$\frac{Q_R}{2D\beta^3} = (C_2 - C_3)\cos\frac{\beta l}{2}\cdot\cosh\frac{\beta l}{2} - (C_4 + C_1)\sin\frac{\beta l}{2}\cdot\cosh\frac{\beta l}{2} + (C_4 - C_1)\cos\frac{\beta l}{2}\cdot\sinh\frac{\beta l}{2} - (C_2 + C_3)\sin\frac{\beta l}{2}\cdot\sinh\frac{\beta l}{2}$$

[A.8]

$$\frac{M_L}{2D\beta^2} = C_4\cos\frac{\beta l}{2}\cdot\cosh\frac{\beta l}{2} + C_3\sin\frac{\beta l}{2}\cdot\cosh\frac{\beta l}{2} - C_2\cos\frac{\beta l}{2}\cdot\sinh\frac{\beta l}{2} - C_1\sin\frac{\beta l}{2}\cdot\sinh\frac{\beta l}{2}$$

$$\frac{-Q_L}{2D\beta^3} = (C_2 - C_3)\cos\frac{\beta l}{2}\cdot\cosh\frac{\beta l}{2} + (C_4 + C_1)\sin\frac{\beta l}{2}\cdot\cosh\frac{\beta l}{2} - (C_4 - C_1)\cos\frac{\beta l}{2}\cdot\sinh\frac{\beta l}{2} - (C_2 + C_3)\sin\frac{\beta l}{2}\cdot\sinh\frac{\beta l}{2}$$

Solving Equations [A.8] simultaneously gives the following expressions for the four integration constants C_1, C_2, C_3, and C_4:

$$C_1(\sinh\beta l + \sin\beta l) = -\frac{(Q_R + Q_L)}{2D\beta^3}\cos\frac{\beta l}{2}\cdot\cosh\frac{\beta l}{2} + \frac{(M_R + M_L)}{2D\beta^2}\left(\cos\frac{\beta l}{2}\cdot\sinh\frac{\beta l}{2} - \sin\frac{\beta l}{2}\cdot\cosh\frac{\beta l}{2}\right)$$

$$C_4(\sinh\beta l + \sin\beta l) = -\frac{(Q_R + Q_L)}{2D\beta^3}\sin\frac{\beta l}{2}\cdot\sinh\frac{\beta l}{2} + \frac{(M_R + M_L)}{2D\beta^2}\left(\cos\frac{\beta l}{2}\cdot\sinh\frac{\beta l}{2} + \sin\frac{\beta l}{2}\cdot\cosh\frac{\beta l}{2}\right)$$

[A.9]

$$C_2(\sinh\beta l - \sin\beta l) = -\frac{(Q_R - Q_L)}{2D\beta^3}\sin\frac{\beta l}{2}\cdot\cosh\frac{\beta l}{2} - \frac{(M_L - M_R)}{2D\beta^2}\left(\cos\frac{\beta l}{2}\cdot\cosh\frac{\beta l}{2} + \sin\frac{\beta l}{2}\cdot\sinh\frac{\beta l}{2}\right)$$

$$C_3(\sinh\beta l - \sin\beta l) = -\frac{(Q_R - Q_L)}{2D\beta^3}\cos\frac{\beta l}{2}\cdot\sinh\frac{\beta l}{2} - \frac{(M_L - M_R)}{2D\beta^2}\left(\cos\frac{\beta l}{2}\cdot\cosh\frac{\beta l}{2} - \sin\frac{\beta l}{2}\cdot\sinh\frac{\beta l}{2}\right)$$

If Equations [A.9] for the integration constants are substituted into the deflection function, Equation [A.3], and the resulting expression is then evaluated at the two edges of the shell element, i.e., at $x = \pm \frac{l}{2}$ for the right and left edges, respectively (see Figure 7), the following equation is obtained:

$$
\begin{aligned}
(\sinh^2 \beta l - \sin^2 \beta l)\,[w_b]_{x=\pm\frac{l}{2}} = &- \frac{(Q_R + Q_L)}{4D\beta^3}(\cosh \beta l + \cos \beta l)(\sinh \beta l - \sin \beta l) \\
&\mp \frac{(Q_R - Q_L)}{4D\beta^3}(\cosh \beta l - \cos \beta l)(\sinh \beta l + \sin \beta l) \\
&+ \frac{(M_R + M_L)}{4D\beta^2}(\sinh \beta l - \sin \beta l)^2 \\
&\mp \frac{(M_L - M_R)}{4D\beta^2}(\sinh \beta l + \sin \beta l)^2
\end{aligned}
\qquad [A.10]
$$

The bending component of the total deflection at the right (R) edge, i.e., at $x = + \frac{l}{2}$, is then expressed in terms of the applied edge shears and edge moments by the following:

$$
\begin{aligned}
[w_b]_R = &- \frac{Q_R}{2D\beta^3}\left(\frac{\cosh \beta l \cdot \sinh \beta l - \cos \beta l \cdot \sin \beta l}{\sinh^2 \beta l - \sin^2 \beta l}\right) \\
&+ \frac{Q_L}{2D\beta^3}\left(\frac{\cosh \beta l \cdot \sin \beta l - \sinh \beta l \cdot \cos \beta l}{\sinh^2 \beta l - \sin^2 \beta l}\right) \\
&+ \frac{M_R}{2D\beta^2}\left(\frac{\sinh^2 \beta l + \sin^2 \beta l}{\sinh^2 \beta l - \sin^2 \beta l}\right) \\
&- \frac{M_L}{2D\beta^2}\left(\frac{2 \sinh \beta l \cdot \sin \beta l}{\sinh^2 \beta l - \sin^2 \beta l}\right)
\end{aligned}
\qquad [A.11]
$$

The bending component of the total deflection at the left (L) edge, i.e., at $x = -\frac{l}{2}$, is then expressed in terms of the applied edge shears and edge moments by the following:

$$
\begin{aligned}
[w_b]_L = & -\frac{Q_L}{2D\beta^3}\left(\frac{\cosh\beta l\cdot\sinh\beta l - \cos\beta l\cdot\sin\beta l}{\sinh^2\beta l - \sin^2\beta l}\right) \\
& + \frac{Q_R}{2D\beta^3}\left(\frac{\cosh\beta l\cdot\sin\beta l - \sinh\beta l\cdot\cos\beta l}{\sinh^2\beta l - \sin^2\beta l}\right) \\
& + \frac{M_L}{2D\beta^2}\left(\frac{\sinh^2\beta l + \sin^2\beta l}{\sinh^2\beta l - \sin^2\beta l}\right) \\
& - \frac{M_R}{2D\beta^2}\left(\frac{2\sinh\beta l\cdot\sin\beta l}{\sinh^2\beta l - \sin^2\beta l}\right)
\end{aligned}
\qquad [A.12]
$$

Comparing the terms of Equations [A.11] and [A.12] with the corresponding terms of Equation [1] and the appropriate edge coefficients, Equations [7a], shows that

$$
\begin{aligned}
\Lambda^{[3]}(\beta l) &= \frac{\cosh\beta l\cdot\sinh\beta l - \cos\beta l\cdot\sin\beta l}{\sinh^2\beta l - \sin^2\beta l} \\
\Lambda^{[5]}(\beta l) &= \frac{\cosh\beta l\cdot\sin\beta l - \sinh\beta l\cdot\cos\beta l}{\sinh^2\beta l - \sin^2\beta l} \\
\Lambda^{[1]}(\beta l) &= \frac{\sinh^2\beta l + \sin^2\beta l}{\sinh^2\beta l - \sin^2\beta l} \\
\Lambda^{[4]}(\beta l) &= \frac{2\sinh\beta l\cdot\sin\beta l}{\sinh^2\beta l - \sin^2\beta l}
\end{aligned}
\qquad [A.13]
$$

The other two lambda functions, namely, $\Lambda^{[2]}$ and $\Lambda^{[6]}$, enter into the equations for the edge rotations of the shell element, and expressions for these two functions are derived next.

When Equations [A.9] are substituted into the first derivative or slope function, Equation [A.4], the slopes at the two edges of the shell element, i.e., at $x = \pm\frac{l}{2}$ for the right and left edges, respectively (see Figure 7), are given by:

$$(\sinh^2 \beta l - \sin^2 \beta l)\left[\frac{dw_b}{dx}\right]_{x=\pm\frac{l}{2}} = -\frac{(Q_R - Q_L)}{4D\beta^2}(\sinh \beta l + \sin \beta l)^2 \mp \frac{(Q_R + Q_L)}{4D\beta^2}(\sinh \beta l - \sin \beta l)^2 +$$

$$-\frac{(M_L - M_R)}{2D\beta}(\cosh \beta l + \cos \beta l)(\sinh \beta l + \sin \beta l) + \qquad [A.14]$$

$$\pm \frac{(M_R + M_L)}{2D\beta}(\cosh \beta l - \cos \beta l)(\sinh \beta l - \sin \beta l)$$

For those terms in Equation [A.14] which have the double signs, it is intended that the upper sign apply to the right edge and the lower one to the left edge.

Thus the rotation at the right (R) edge, i.e., at $x = +\frac{l}{2}$, is expressed in terms of the applied edge shears and edge moments by the following:

$$-\left[\frac{dw_b}{dx}\right]_{x=+\frac{l}{2}} \equiv +\theta_R = +\frac{Q_R}{2D\beta^2}\left(\frac{\sinh^2 \beta l + \sin^2 \beta l}{\sinh^2 \beta l - \sin^2 \beta l}\right) - \frac{Q_L}{2D\beta^2}\left(\frac{2\sinh \beta l \cdot \sin \beta l}{\sinh^2 \beta l - \sin^2 \beta l}\right) +$$

$$-\frac{M_R}{D\beta}\left(\frac{\cosh \beta l \cdot \sinh \beta l + \cos \beta l \cdot \sin \beta l}{\sinh^2 \beta l - \sin^2 \beta l}\right) + \qquad [A.15]$$

$$+\frac{M_L}{D\beta}\left(\frac{\cosh \beta l \cdot \sin \beta l + \sinh \beta l \cdot \cos \beta l}{\sinh^2 \beta l - \sin^2 \beta l}\right)$$

and the rotation at the left (L) edge, i.e., at $x = -\frac{l}{2}$, is expressed in terms of the applied edge shears and edge moments by the following:

$$+\left[\frac{dw_b}{dx}\right]_{x=-\frac{l}{2}} \equiv +\theta_L = +\frac{Q_L}{2D\beta^2}\left(\frac{\sinh^2 \beta l + \sin^2 \beta l}{\sinh^2 \beta l - \sin^2 \beta l}\right) - \frac{Q_R}{2D\beta^2}\left(\frac{2\sinh \beta l \cdot \sin \beta l}{\sinh^2 \beta l - \sin^2 \beta l}\right) +$$

$$-\frac{M_L}{D\beta}\left(\frac{\cosh \beta l \cdot \sinh \beta l + \cos \beta l \cdot \sin \beta l}{\sinh^2 \beta l - \sin^2 \beta l}\right) + \qquad [A.16]$$

$$+\frac{M_R}{D\beta}\left(\frac{\cosh\beta l\cdot\sin\beta l+\cos\beta l\cdot\sinh\beta l}{\sinh^2\beta l-\sin^2\beta l}\right)$$

Comparing the terms of Equations [A.15] and [A.16] with the corresponding terms of Equation [2] and the appropriate edge coefficients, Equations [7a], shows that the remaining two lambda functions, besides those defined by [A.13], are given by:

$$\Lambda^{[2]}(\beta l)=\frac{\cosh\beta l\cdot\sinh\beta l+\cos\beta l\cdot\sin\beta l}{\sinh^2\beta l-\sin^2\beta l}$$

$$\Lambda^{[6]}(\beta l)=\frac{\cosh\beta l\cdot\sin\beta l+\sinh\beta l\cdot\cos\beta l}{\sinh^2\beta l-\sin^2\beta l} \qquad [A.17]$$

Consequently, the set of functions defined by Equations [A.13] and [A.17] are exactly those given as Equations [8] earlier in the report.

APPENDIX B

DEVELOPMENT OF EDGE COEFFICIENTS AND EXPRESSIONS FOR THE RADIAL AND TANGENTIAL STRESSES OF A CIRCULAR ANNULUS

With reference to Equation [5] and Figure 5, it has been assumed that the edge moments M^A, axial thrusts P^A, and surface pressure p do not give rise to any radial displacements w^A in the plane of the circular annulus. Only the inplane forces H_i^A and H_j^A acting on the outer and inner circular boundaries, respectively, of the annulus give rise to such deformations.

On Page 418 of Reference 6 the following expression is given, based on the Lamé or plane-strain solution for a thick-walled tube subjected to simultaneous internal pressure p_i and external pressure p_0:

$$\overline{w}(r) = \frac{r^2 (1-\nu)(p_i r_i^2 - p_0 r_0^2) + (1+\nu)(p_i - p_0) r_i^2 r_0^2}{rE\,(r_0^2 - r_i^2)} \qquad \text{[B.1]}$$

where r_i and r_0 are the radii to the inside and outside circular boundaries, respectively, of the tube, and p_i and p_0 are the radial pressures acting on the inside and outside surfaces, respectively, of the tube. The variable "r" is the radial distance from the axis of the tube to a point in question through the thickness of the tube wall.

Adapting the solution, Equation [B.1], to the present problem of the circular annulus, we see that

$$r_0 = \overline{R}_i;\; r_i = \overline{R}_j$$

$$p_0 = +\frac{H_i^A}{t}\;;\; p_i = -\frac{H_j^A}{t} \qquad \text{[B.2]}$$

Substituting [B.2] into [B.1] and adapting the sign convention of Figure 5 for positive radial displacement, we obtain the following results:

$$-\overline{w}(r) = -\frac{r\,(1-\nu)(H_j^A \overline{R}_j^2 + H_i^A \overline{R}_i^2) + (1+\nu)(H_j^A + H_i^A)\overline{R}_i^2 \overline{R}_j^2}{rE^A t\,(\overline{R}_i^2 - \overline{R}_j^2)} \qquad \text{[B.3]}$$

To find the edge coefficients g_i^A, g_{ij}^A, g_j^A, and g_{ji}^A appearing in Equation [5] and its counterpart in which $i \rightarrow j$ and $j \rightarrow i$, it is only necessary to substitute the following successive four conditions into the basic solution, Equation [B.3]:

$$\text{To get } g_i^A : \quad \text{set } r = \bar{R}_i ; \; H_j^A = 0 ; \; H_i^A = 1 \qquad [B.\ 4]$$

$$g_{ij}^A : \quad r = \bar{R}_i ; \; H_j^A = 1 ; \; H_i^A = 0 \qquad [B.\ 5]$$

$$g_j^A : \quad r = \bar{R}_j ; \; H_j^A = 1 ; \; H_i^A = 0 \qquad [B.\ 6]$$

$$g_{ji}^A : \quad r = \bar{R}_j ; \; H_j^A = 0 ; \; H_i^A = 1 \qquad [B.\ 7]$$

Thus conditions [B.4], [B.5], [B.6], and [B.7] when substituted into Equation [B.3] lead to the following equations, respectively:

$$g_i^A = \frac{(1+\nu)\bar{R}_i^2\bar{R}_j^2}{E^A t(\bar{R}_i^2 - \bar{R}_j^2)} \left[\frac{1}{\bar{R}_i} + \left(\frac{1-\nu}{1+\nu} \right) \frac{\bar{R}_i}{\bar{R}_j^2} \right] \qquad [B.\ 8]$$

$$g_{ij}^A = \frac{(1+\nu)\bar{R}_i^2\bar{R}_j^2}{E^A t(\bar{R}_i^2 - \bar{R}_j^2)} \left[\left(\frac{2}{1+\nu} \right) \frac{1}{\bar{R}_i} \right] \qquad [B.\ 9]$$

$$g_j^A = \frac{(1+\nu)\bar{R}_i^2\bar{R}_j^2}{E^A t(\bar{R}_i^2 - \bar{R}_j^2)} \left[\frac{1}{\bar{R}_j} + \left(\frac{1-\nu}{1+\nu} \right) \frac{\bar{R}_j}{\bar{R}_i^2} \right] \qquad [B.10]$$

$$g_{ji}^A = \frac{(1+\nu)\bar{R}_i^2\bar{R}_j^2}{E^A t(\bar{R}_i^2 - \bar{R}_j^2)} \left[\left(\frac{2}{1+\nu} \right) \frac{1}{\bar{R}_j} \right] \qquad [B.11]$$

With $i \to 1$, $j \to 2$, and $t = h_3$, Equations [B.8], [B.9], [B.10], and [B.11] become exactly Equations [10a], respectively.

For expressions [6] for the edge rotations of the annulus, symmetry and loading conditions for a typical bay of the web-stiffened sandwich cylinder far removed from end effects dictate that these rotations not only total zero but each and every component is zero. The more general case shown in Figure 5 and reflected by Equations [6] will be considered in a separate report.

Equations [18] and [19] for the radial and tangential stresses, respectively, in the circular annulus are derived next by following the solution given on Pages 415 to 418 of Reference 6. The plane-strain theory applied to the axisymmetric elastic deflections of a thick-walled tube results in the following differential equation:

$$\frac{d^2u}{dr^2} + \frac{1}{r}\frac{du}{dr} - \frac{u}{r^2} = 0 \qquad [B.12]$$

where u is the radial displacement at a point in the wall of the tube at a radial distance "r" away from the axis of the tube. The solution of Equation [B.12] is given by:

$$u(r) = Ar + \frac{B}{r} \qquad \text{[B.13]}$$

The integration constants "A" and "B" are determined from the following deflection boundary conditions:

$$\begin{aligned} \text{at } r = \bar{R}_j:\ u &= -\,w_j^A \\ r = \bar{R}_i:\ u &= -\,w_i^A \end{aligned} \qquad \text{[B.14]}$$

Substituting the conditions [B.14] into the solution [B.13] gives:

$$\begin{aligned} A &= -\left(\frac{w_i^A \bar{R}_i - w_j^A \bar{R}_j}{\bar{R}_i^2 - \bar{R}_j^2}\right) \\ B &= -\,\bar{R}_i \bar{R}_j \left(\frac{w_j^A \bar{R}_i - w_i^A \bar{R}_j}{\bar{R}_i^2 - \bar{R}_j^2}\right) \end{aligned} \qquad \text{[B.15]}$$

In Reference 6 the radial and tangential stresses, respectively, as a function of the distance "r," are as follows:

$$\sigma_r = \frac{E^A}{(1-\nu^2)}\left[\frac{du}{dr} + \nu\,\frac{u}{r}\right] \qquad \text{[B.16]}$$

$$\sigma_t = \frac{E^A}{(1-\nu^2)}\left[\frac{u}{r} + \nu\,\frac{du}{dr}\right] \qquad \text{[B.17]}$$

Substituting the deflection $u\,(r)$ and its first derivative $\frac{du\,(r)}{dr}$ from Equation [B.13] into Equations [B.16] and [B.17] yields:

$$\sigma_r = E^A\left[\frac{A}{(1-\nu)} - \frac{B}{(1+\nu)}\cdot\frac{1}{r^2}\right] \qquad \text{[B.18]}$$

$$\sigma_t = E^A\left[\frac{A}{(1-\nu)} + \frac{B}{(1+\nu)}\cdot\frac{1}{r^2}\right] \qquad \text{[B.19]}$$

Note

$$\sigma_r + \sigma_t = 2E^A \frac{A}{(1-\nu)} = \text{constant} \qquad [B.20]$$

which is a consequence of the plane-strain assumption, i.e.,

$$\epsilon_z = -\frac{\nu}{E}(\sigma_r + \sigma_t) = \text{constant} \qquad [B.21]$$

Equation [B.21] results by putting the axial stress σ_z equal to zero in the three-dimensional Hooke's law.

The maximum radial stress $\sigma_{r_{max}}$ occurs on the outer boundary of the annulus, i.e., at $r = \bar{R}_i$, whereas the maximum tangential stress $\sigma_{t_{max}}$ occurs on the inner boundary, i.e., at $r = \bar{R}_j$. This together with $i \to 1$ and $j \to 2$ results in Equations [18] and [19].

APPENDIX C

DEVELOPMENT OF EXPRESSIONS FOR THE SHELL STRESSES, EQUATIONS [11] THROUGH [14]

In Reference 7 Salerno and Pulos developed a theory for the axisymmetric elastic deformations and stresses in a ring-stiffened, perfectly circular cylindrical shell subjected to uniform external hydrostatic pressure. Equations developed by these authors for the critical shell stresses are reviewed here and adapted to the present problem of the two coaxial cylinder elements comprising the web-stiffened sandwich cylinder structure; see Figure 1.

From symmetry considerations (Figure 4), the general solution for the bending deformations, i.e., Equation [A.3], simplifies to:

$$w_b(x) = C_1 \cos\beta x \cdot \cosh\beta x + C_4 \sin\beta x \cdot \sinh\beta x \qquad \text{[C.1]}$$

The particular integrals to the differential Equation [A.1], which constitute the membrane deformations and which must be added to the bending component [C.1] to get the total deflection, are given by Equations [16] for the outside and inside cylindrical shells, respectively. The loading condition to which Equations [16] apply is shown in Figure 1. The total deflection can thus be written in the following form to apply to both cylinders:

$$w(x) \equiv w_b(x) + w^p = C_1 \cos\beta x \cdot \cosh\beta x + C_4 \sin\beta x \cdot \sinh\beta x + w^p \qquad \text{[C.2]}$$

The first derivative or slope expression is then given by:

$$\frac{dw(x)}{dx} = (C_1 + C_4)\beta \cos\beta x \cdot \sinh\beta x - (C_1 - C_4)\beta \sin\beta x \cdot \cosh\beta x \qquad \text{[C.3]}$$

The integration constants C_1 and C_4 are determined from the following deformation boundary conditions:

$$\text{at } x = \pm\frac{l}{2} \;:\; w_i = w_i^s \,;\; \frac{dw_i}{dx} \equiv \mp\, \theta_i^s = 0 \qquad \text{[C.4]}$$

When the conditions [C.4] are substituted into Equations [C.2] and [C.3], the constants C_1 and C_4 are found to be:

$$C_1 = +(w^s - w^p)\left(\frac{\cos\frac{\beta l}{2}\cdot\sinh\frac{\beta l}{2} + \sin\frac{\beta l}{2}\cdot\cosh\frac{\beta l}{2}}{\cosh\frac{\beta l}{2}\cdot\sinh\frac{\beta l}{2} + \sin\frac{\beta l}{2}\cdot\cos\frac{\beta l}{2}}\right) \equiv -(w^p - w^s)\ \frac{\Lambda^{[6]}\left(\frac{\beta l}{2}\right)}{\Lambda^{[2]}\left(\frac{\beta l}{2}\right)}$$

[C.5]

$$C_4 = -(w^s - w^p)\left(\frac{\cos\frac{\beta l}{2}\cdot\sinh\frac{\beta l}{2} - \sin\frac{\beta l}{2}\cdot\cosh\frac{\beta l}{2}}{\cosh\frac{\beta l}{2}\cdot\sinh\frac{\beta l}{2} + \sin\frac{\beta l}{2}\cdot\cos\frac{\beta l}{2}}\right) \equiv -(w^p - w^s)\ \frac{\Lambda^{[5]}\left(\frac{\beta l}{2}\right)}{\Lambda^{[2]}\left(\frac{\beta l}{2}\right)}$$

where the lambda functions $\Lambda^{[2]}$, $\Lambda^{[5]}$, and $\Lambda^{[6]}$ are defined by Equations [8].

The principal stresses in the longitudinal and circumferential directions of the shell elements are given by the following expressions, respectively:

$$\sigma_X = -\frac{P}{h} + \sigma_{xb} \qquad [C.6]$$

$$\sigma_\Phi = -E\frac{w}{R} - \nu\frac{P}{h} + \nu\sigma_{xb} \qquad [C.7]$$

where the first term in Equation [C.6] and the first two terms in Equation [C.7] are the corresponding membrane stress components and the remaining terms are the bending components. In terms of the shell curvatures, the stress expressions [C.6] and [C.7] become (see, for example, Reference 7):

$$\sigma_X(x) = -\frac{P}{h} \pm \frac{Eh}{2(1-\nu^2)}\cdot\frac{d^2w(x)}{dx^2} \qquad [C.8]$$

$$\sigma_\Phi(x) = -E\frac{w(x)}{R} - \nu\frac{P}{h} \pm \frac{\nu Eh}{2(1-\nu^2)}\cdot\frac{d^2w(x)}{dx^2} \qquad [C.9]$$

Substituting the deflection $w(x)$ and the second derivative of $w(x)$ from Equation [C.2] into the above, gives the following equations:

$$\sigma_X(x) = -\frac{P}{h} \pm \frac{Eh\beta^2}{(1-\nu^2)} \left[-C_1 \sin\beta x \cdot \sinh\beta x + C_4 \cos\beta x \cdot \cosh\beta x\right] \quad [C.10]$$

$$\begin{aligned}\sigma_\Phi(x) = -\frac{E}{R} w^p - \nu \frac{P}{h} + E &\left[-\frac{C_1}{R} \pm \frac{\nu h\beta^2}{(1-\nu^2)} C_4\right] \cos\beta x \cdot \cosh\beta x + \\ -E &\left[\frac{C_4}{R} \pm \frac{\nu h\beta^2}{(1-\nu^2)} C_1\right] \sin\beta x \cdot \sinh\beta x\end{aligned} \quad [C.11]$$

Once the constants C_1 and C_4 as given by Equations [C.5] are substituted into Equations [C.10] and [C.11], the distributions of total longitudinal and total circumferential stress between adjacent supporting elements, imposing the restraint conditions defined by [C.4] on the shell "edges," can then be determined.

Of particular interest are the critical stresses that occur at a point between adjacent supporting elements, i.e., at $x = 0$, and immediately at a supporting element, i.e., at $x = \pm \frac{l}{2}$; see Figure 4. These critical stresses are found from Equations [C.10] and [C.11] to be:

AT MIDBAY ($x = 0$):

$$\sigma_{Xm} = -\frac{P}{h} \pm \frac{Eh\beta^2}{(1-\nu^2)} C_4 \quad [C.12]$$

$$\sigma_{\Phi m} = -\frac{E}{R} w^p - \nu \frac{P}{h} + E\left[-\frac{C_1}{R} \pm \frac{\nu h\beta^2}{(1-\nu^2)} C_4\right] \quad [C.13]$$

AT A SUPPORT ($x = \pm \frac{l}{2}$):

$$\sigma_{Xf} = -\frac{P}{h} \pm \frac{Eh\beta^2}{(1-\nu^2)} \left[-C_1 \sin\frac{\beta l}{2} \cdot \sinh\frac{\beta l}{2} + C_4 \cos\frac{\beta l}{2} \cdot \cosh\frac{\beta l}{2}\right] \quad [C.14]$$

$$\sigma_{\Phi f} = -\frac{E}{R} w^p - \nu \frac{P}{h} + E \left[-\frac{C_1}{R} \pm \frac{\nu h \beta^2}{(1-\nu^2)} C_4 \right] \cos \frac{\beta l}{2} \cdot \cosh \frac{\beta l}{2} + \qquad \text{[C.15]}$$

$$-E \left[\frac{C_4}{R} \pm \frac{\nu h \beta^2}{(1-\nu^2)} C_1 \right] \sin \frac{\beta l}{2} \cdot \sinh \frac{\beta l}{2}$$

The substitution of Equations [C.5] for the constants C_1 and C_4 into Equations [C.12] through [C.15], and the introduction of the lambda functions defined by Equations [8] into the resulting expressions lead to Equations [11] through [14], respectively, for the critical shell stresses.

The total deflections at midbay, i.e., $x = 0$, and at a web stiffener, i.e., $x = \frac{l}{2}$, can be found by using Equations [C.2] and [C.5]. These are, respectively:

$$w(0) = -(w^p - w^s) \frac{\Lambda^{[6]}\left(\frac{\beta l}{2}\right)}{\Lambda^{[2]}\left(\frac{\beta l}{2}\right)} + w^p \qquad \text{[C.16]}$$

$$w\left(\frac{l}{2}\right) = -\frac{(w^p - w^s)}{\Lambda^{[2]}\left(\frac{\beta l}{2}\right)} \left[\Lambda^{[6]}\left(\frac{\beta l}{2}\right) \cos \frac{\beta l}{2} \cdot \cosh \frac{\beta l}{2} \right. \qquad \text{[C.17]}$$

$$\left. + \Lambda^{[5]}\left(\frac{\beta l}{2}\right) \sin \frac{\beta l}{2} \cdot \sinh \frac{\beta l}{2} \right] + w^p$$

APPENDIX D

DETERMINATION OF AXIAL-PRESSURE LOAD DISTRIBUTION TO THE TWO COAXIAL CYLINDERS

With reference to Figure 8, if it is assumed that the web elements do not resist any axial load, then force equilibrium in the longitudinal direction requires that

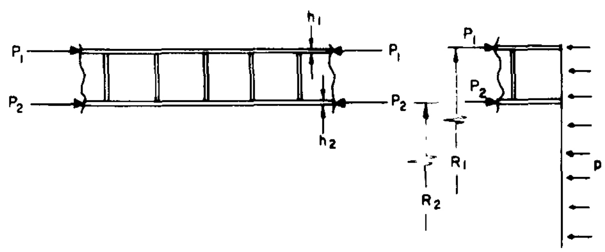


Figure 8 – Distribution of Axial-Pressure Load to the Two Cylindrical Shells

$$P_1 R_1 + P_2 R_2 = \frac{pR_1^2}{2} \qquad \text{[D.1]}$$

where the axial stress forces P_1 and P_2 in the outer and inner cylindrical shells, respectively, are the unknown quantities to be determined. To find explicit expressions for P_1 and P_2 another relationship between these quantities, the applied pressure p, and the geometry of the shells is needed.

If it is assumed that both cylindrical shells displace the same amount longitudinally, i.e.,

$$u_1 = u_2 \qquad \text{[D.2]}$$

then the integral of the longitudinal midthickness strains over a stiffener spacing for each of the two cylindrical shells must be equal. Therefore,

$$\int_0^{l/2} (\epsilon_{xM})_1 \, dx = \int_0^{l/2} (\epsilon_{xM})_2 \, dx \qquad \text{[D.3]}$$

Equation [D.3] is a consequence of the strain-displacement relation

$$\epsilon_x = \frac{du}{dx} \qquad \text{[D.4]}$$

Introducing the two-dimensional Hooke's law,

$$\epsilon_x = \frac{1}{E}\,(\sigma_x - \nu\sigma_\phi)$$

$$\epsilon_\phi = \frac{1}{E}\,(\sigma_\phi - \nu\sigma_x) \qquad \text{[D.5]}$$

into Equation [D.3], we obtain

$$\int_0^{l/2} [(1-\nu^2)\,\sigma_{xM} - \nu E^s\,\epsilon_{\phi M}]_1\,dx = \int_0^{l/2} [(1-\nu^2)\,\sigma_{xM} - \nu E^s\,\epsilon_{\phi M}]_2\,dx \qquad \text{[D.6]}$$

Since the longitudinal membrane stress and the circumferential membrane strain in each of the two shells are given, respectively, by:

$$\sigma_{xM} = \frac{P}{h} \qquad \text{[D.7]}$$

$$\epsilon_{\phi M} = \frac{w}{R} \qquad \text{[D.8]}$$

then Equation [D.6] becomes:

$$\int_0^{l/2} \left[\frac{(1-\nu^2)\,P_1}{E^s h_1} - \nu\,\frac{w_1(x)}{R_1}\right] dx = \int_0^{l/2} \left[\frac{(1-\nu^2)\,P_2}{E^s h_2} - \nu\,\frac{w_2(x)}{R_2}\right] dx \qquad \text{[D.9]}$$

Substituting the deflection function [C.2] together with the appropriate expressions for the membrane deflections w_i^p for each of the two shells from Equations [16] into Equation [D.9], carrying out the indicated integrations, and finally introducing Equations [24], [22j, and [21] into the resulting expression, we obtain

$$
\frac{P_1}{h_1}\left[1-\nu^2\,\frac{2}{\beta_1 l}\,\frac{\Lambda^{[1]}\left(\frac{\beta_1 l}{2}\right)}{\Lambda^{[2]}\left(\frac{\beta_1 l}{2}\right)}\right]-\frac{\nu E^s}{R_1}\,\frac{2}{\beta_1 l}\,(g_1^A H_1+\bar{g}_1^A H_2)\,\frac{\Lambda^{[1]}\left(\frac{\beta_1 l}{2}\right)}{\Lambda^{[2]}\left(\frac{\beta_1 l}{2}\right)}
$$

$$
-\nu\,\frac{pR_1}{h_1}\left[1-\frac{2}{\beta_1 l}\,\frac{\Lambda^{[1]}\left(\frac{\beta_1 l}{2}\right)}{\Lambda^{[2]}\left(\frac{\beta_1 l}{2}\right)}\right]
$$

$$
=\frac{P_2}{h_2}\left[1-\nu^2\,\frac{2}{\beta_2 l}\,\frac{\Lambda^{[1]}\left(\frac{\beta_2 l}{2}\right)}{\Lambda^{[2]}\left(\frac{\beta_2 l}{2}\right)}\right]
$$

$$
-\frac{\nu E^s}{R_2}\,\frac{2}{\beta_2 l}\,(g_2^A H_2+\bar{g}_2^A H_1)\,\frac{\Lambda^{[1]}\left(\frac{\beta_2 l}{2}\right)}{\Lambda^{[2]}\left(\frac{\beta_2 l}{2}\right)} \qquad \text{[D.10]}
$$

Equations [D.1] and [D.10] constitute two equations in the two unknown forces P_1 and P_2; when they are solved simultaneously, the following expressions are found:

$$
P_1=\frac{pR_1\left[\nu(1-a_1)+\frac{1}{2}\,\frac{R_1h_1}{R_2h_2}\,(1-\nu^2 a_2)\right]+\frac{\nu E^s h_1}{R_1}\,(g_1^A H_1+\bar{g}_1^A H_2)\,a_1-\frac{\nu E^s h_1}{R_2}\,(g_2^A H_2+\bar{g}_2^A H_1)\,a_2}{1-\nu^2 a_1+\frac{R_1h_1}{R_2h_2}\,(1-\nu^2 a_2)} \qquad \text{[D.11]}
$$

$$
P_2=\frac{pR_1\left[-\nu(1-a_1)\,\frac{R_1}{R_2}+\frac{1}{2}\,\frac{R_1}{R_2}\,(1-\nu^2 a_1)\right]-\frac{\nu E^s h_1}{R_2}\,(g_1^A H_1+\bar{g}_1^A H_2)\,a_1+\frac{\nu E^s R_1 h_1}{R_2^2}\,(g_2^A H_2+\bar{g}_2^A H_1)\,a_2}{1-\nu^2 a_1+\frac{R_1h_1}{R_2h_2}\,(1-\nu^2 a_2)} \qquad \text{[D.12]}
$$

where

$$a_1 = \frac{2}{\beta_1 l} \frac{\Lambda^{[1]}\left(\frac{\beta_1 l}{2}\right)}{\Lambda^{[2]}\left(\frac{\beta_1 l}{2}\right)}; \quad a_2 = \frac{2}{\beta_2 l} \frac{\Lambda^{[1]}\left(\frac{\beta_2 l}{2}\right)}{\Lambda^{[2]}\left(\frac{\beta_2 l}{2}\right)} \qquad [D.13]$$

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Equations are given for computing numerically the longitudinal and circumferential stresses in the two coaxial cylindrical shells and the radial and tangential stresses in the web stiffeners between the two shells.

1. Cylindrical shells (Stiffened)--Stresses--Mathematical analysis
2. Cylindrical shells (Stiffened)--Deformation--Mathematical analysis
3. Cylindrical shells (Stiffened)--Sandwich construction

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